142857

A Full-Circle Story

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The Magic Number 142857

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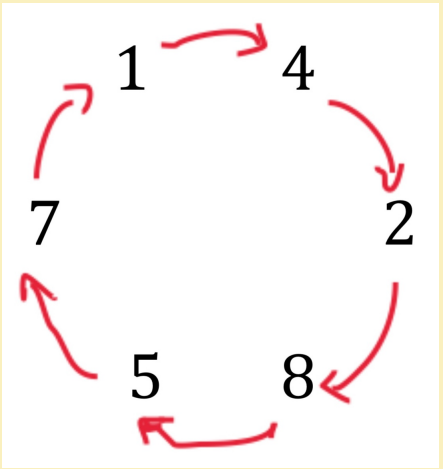

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This will happen when you multiply 142857 by any number that is not a multiple of 7.

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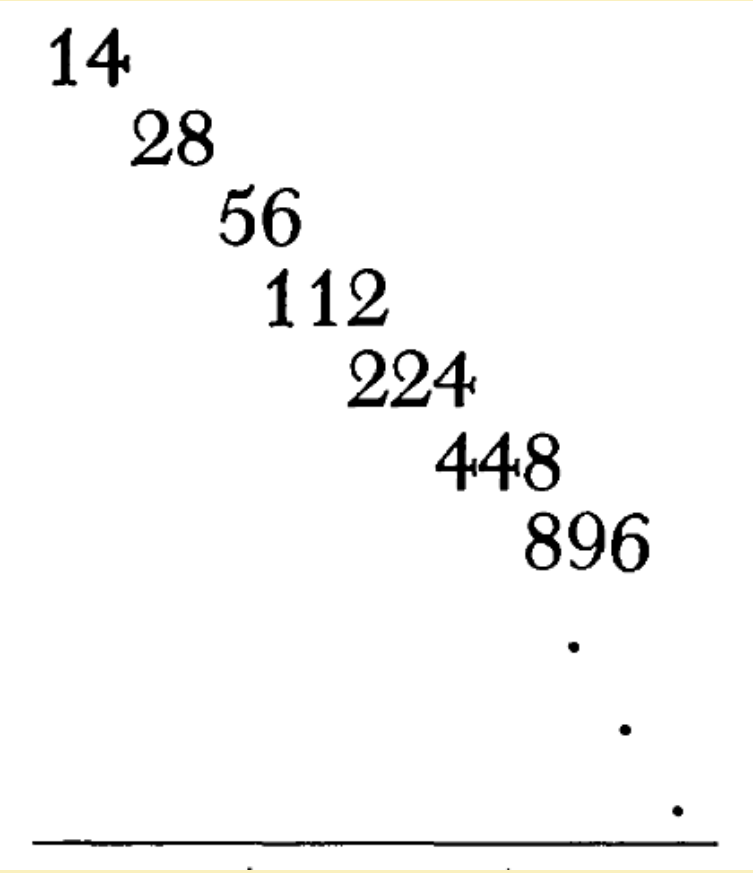

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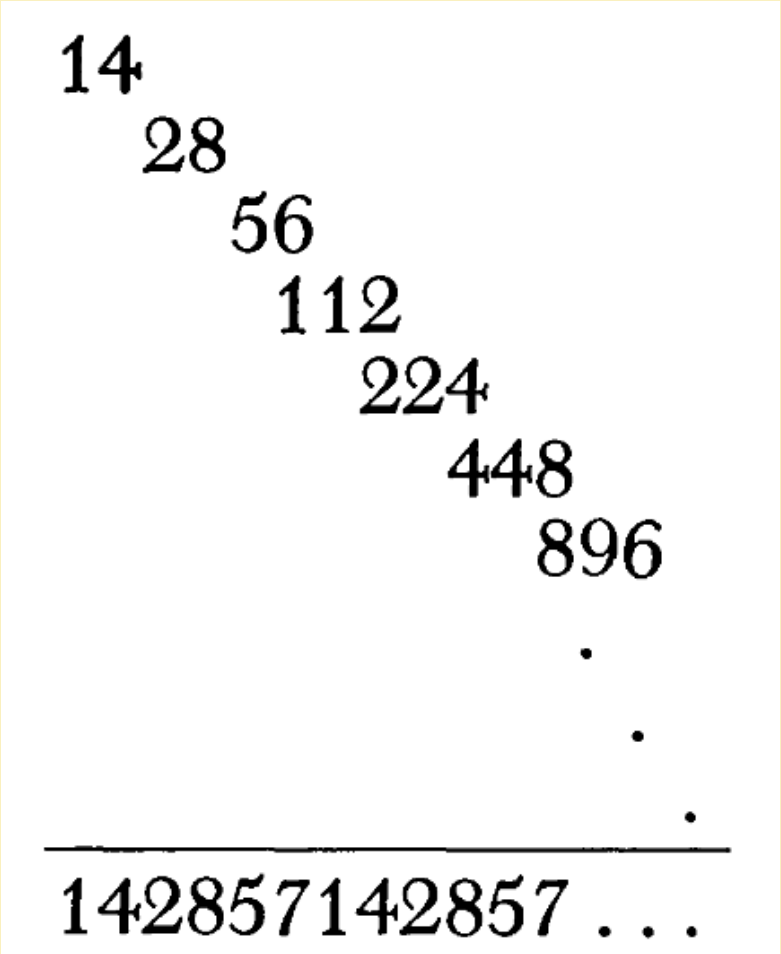

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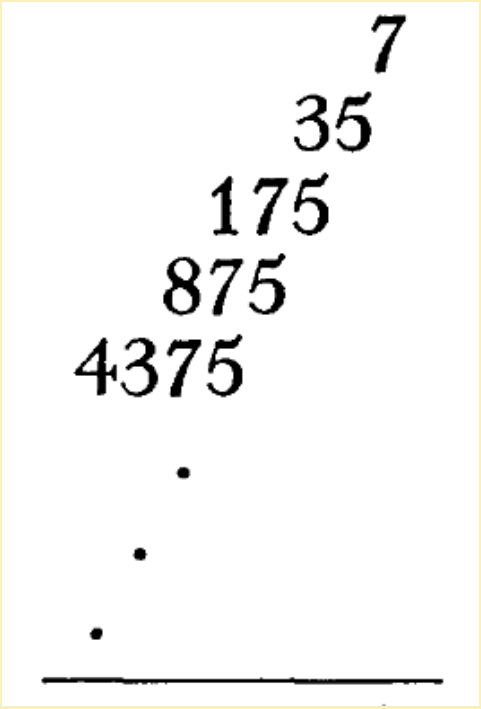

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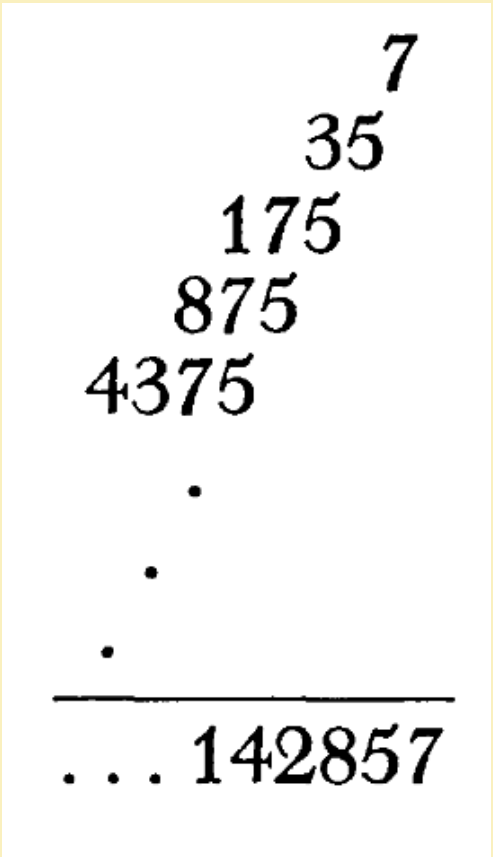

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As 142857 becomes known amongst audiences, it becomes important for applications in magic to be able to use this number's magical properties, without necessarily using this particular number directly.

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Are there other numbers like it?

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These are called cyclic numbers!

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Similarly, the cyclic number 0588235294117647 is obtained by typing $1 \div 17$:

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Note that the repeating block 076923 of $1 \div 13$ is the same size as that of $1 \div 7$.

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- $1 \div 13$ does not yield a cyclic number because the number of digits in the repeating block of $1 \div 13$ has only 6 digits.

Table of the First Few Cyclic Numbers

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Prime (p)	**Cyclic Number (N)**	**Length ($p-1$)**
7	142857	6
17	0588235294117647	16
19	052631578947368421	18
23	0434782608695652173913	22
29	03448275862068965517241379310	28
47	021276595744680851063829787234042553191489361702	46

Table: The first six cyclic numbers.

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The next several primes satisfying the desired property are:

59, 61, 97, 109, 113, 131, 149, 167, 179, 181, 193, 223, 229, 233, 257, ...

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$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p(p-1)}\right) \cdot \frac{N}{\log N} \approx .374 \frac{N}{\log N}.$$

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In particular, there are infinitely many cyclic numbers!

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In other words, half the multiples are cyclic shifts of 076923, and the other half are cyclic shifts of 153846!

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$$\frac{1}{3} \cdot \prod_{p \text{ prime}} \left(1 - \frac{1}{p(p-1)}\right) \cdot \frac{N}{\log N} \approx .124 \frac{N}{\log N}.$$

There were similar conjectures about tri-cyclic numbers, quadri-cyclic numbers, etc.

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Again, we can now prove, using these new methods, that there are an infinite number of each of them!

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Arithmetic statistics is the area of mathematics that studies the statistics, distribution, and asymptotic behavior of natural classes of arithmetic objects (such as cyclic numbers, prime numbers, solutions to equations, number fields, class groups, rational points on varieties, etc.).

The area has seen a rejuvenation in recent years due to an injection of a number of new ideas into the subject, from geometry, analysis, group theory, representation theory of Lie groups, and more.

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Finally, cyclic numbers provide a truly wonderful example of how seemingly recreational questions can often lead to deep connections and applications that could never have been predicted.