Independent Work in the Mathematics Department:
A Guide for Mathematics Majors

Mathematics: A Portrait of the Discipline

The fundamental areas of mathematics, such as analysis, geometry, topology, and algebra, developed in parallel to scientific inquiry into the natural world. To a large extent, mathematics serves as the language and the key analytic tool for theory building in all the natural sciences, as well as in economics and finance. Other areas of mathematics such as number theory and logic arose from more philosophical considerations, but they now are fundamental tools for all mathematicians and for theoreticians in a wide variety of disciplines including philosophy and computer science in addition to the fields listed above.

Math majors at Princeton work with some of the best mathematicians in the world on a wide variety of topics in both pure and applied mathematics. These include analysis, algebra and number theory, geometry, topology, combinatorics and graph theory, probability, and differential equations. Some students focus on pure research in these fields, and others work on topics in applied mathematics. The math major is necessarily broad and flexible, with many students doing their independent work in allied areas including computer science, physics, chemistry, and biology, as well as philosophy. Applications to social sciences, including but not limited to economics and finance, are also common.

Typical math majors are highly critical and creative thinkers with broad intellectual interests in both the humanities and the sciences. They are excited by the clarity, precision, and power of the rigorous and logical thinking they encounter in the proof-based courses that are the prerequisites for the major. As students move through the major in the second and third years, they are introduced to a broad spectrum of mathematical knowledge. In the junior independent work and in the senior thesis, they experience the thrills and the challenges of independent work in mathematics. They will learn how to combine specific mathematical tools, some familiar and some new, with creative and disciplined logical thinking in order to balance the desire for clarity with the uncertainty inherent in research, theory-building and applications.

Goals of Independent Work

Junior independent work in the mathematics department consists of a junior seminar or junior paper (JP) during each term of the junior year. Juniors may do a seminar in each semester, or they may do a seminar in one semester and a paper in the other. Senior independent work consists of a thesis spanning the final year, along with a defense of that thesis.

Building on introductory courses in the first year which emphasize logical reasoning and formal mathematical proofs, students in the second year learn the fundamentals of a broad sampling of well-established mathematical knowledge, using textbooks on analysis, geometry and algebra. As their mathematical skills and interests develop through this survey of the fundamentals, students move into the third year with more freedom to branch out and explore specialized and advanced mathematical topics they find particularly appealing.

The goal of the independent work of the junior year is to develop the skills needed to move beyond textbook-based learning and begin to engage actively with the contemporary research literature. A typical research paper in mathematics will refer to thirty or more papers, and it will assume that the

reader is generally familiar with the standard techniques and problems that are under investigation. The references in such a paper give a reading program which could require many years to master in order to become an expert in this area of research. Juniors and seniors do not have "many" years, only a portion of one or two. The task of a young mathematician is therefore to learn how to navigate through a dizzying blur of established knowledge, to join the scholarly discussion in progress, and gradually orient themselves in a rapidly developing research area. This is a completely new way of doing mathematics compared to the previous class work, and, in the process, students must develop several new skills.

In reading a research paper, a student must decide what is important and new in the paper and how it connects with already familiar mathematical ideas. The student must carefully choose which references to read, and how deeply to read them, in order to make timely progress. Instead of learning everything presented to them as thoroughly as possible, the student must learn to look for the main ideas first. They must learn to be patient and deliberate in the face of uncomfortable uncertainty, building on small nuggets of understanding to reach a full and deeper knowledge. They must fill in gaps in background knowledge judiciously, work out examples of their own design, and open their minds to questions that will not be answered immediately. They must learn to trust their own ability to use these examples and questions to build their own path and to find their own individual point of view as a scholar in the field.

After two years of taking math courses, students have developed a good general knowledge of the main classical areas of mathematics and have found a very successful way of working, appropriate for learning mathematics from textbooks and lectures and problem sets that provide a clear road map and day-to-day focus. By contrast, in independent work the student and adviser construct that road map and focus collaboratively. Typically, the student begins with some broad readings to get oriented in a new landscape. The student reports on those readings to the adviser, and in the subsequent discussions a sharper focus and specific research goals gradually emerge to shape, eventually, the final project. Good student and adviser collaboration is essential to keep the momentum going and help the student produce a clear and convincing exposition of a substantial mathematical investigation in the time frame available.

Students should meet regularly with advisers to talk through what they have been thinking about, even if, and *especially if*, they have no new results to report. These meetings keep the work on track and provide an opportunity for small course corrections and insights that will dramatically increase the pace of progress for most students. Just the process of trying to explain what one has read to another person who is familiar with the problem can help clarify one's thinking in quite dramatic ways. Advisers are teachers as well as researchers, and they really worry when students are not answering emails or showing up for scheduled meetings.

As they explore independently, students will develop the **precision of their logical thinking**—what mathematicians call **rigor**. They will **learn to use the formal logical approach of mathematics** to spot hidden, unwarranted assumptions in an argument and to root out inconsistencies. They will learn **to identify deep structural similarities in problems** that seem unrelated on the surface.

In order to begin work on any topic in contemporary mathematics, finding and exploring the research literature is a necessary but daunting task. The student will work with a variety of sources, and, as understanding grows, the student will **begin to develop their own point of view and their own unique scholarly voice**. With time and effort, the student begins to identify interesting problems they want to investigate and to develop their own ideas about how to carry out those investigations.

Eventually they **learn how to come up with a complete argument of their own**, using ideas and techniques from various sources to build a whole greater than the sum of its parts.

At this point the student is ready to write a paper, or give a talk, explaining their work to other mathematicians. The audience needs to know where the research topic fits in the broad scheme of mathematics. Which mathematicians first developed this topic and its methods? Is this a new field or a field with a long history? How does this work connect to other areas of mathematics? Why is the subject interesting and important? Where is this work heading? From there, the student should introduce the specific topic of their independent work.

Although mathematics is "constructed" out of chains of logic, it does not follow that mathematical writing will automatically be clear. For **clarity of exposition**, students should begin, after a section of motivation, with the appropriate definitions. Basic lemmas and propositions must follow from the definitions, and the main theorems build on those. Students will also learn where to include examples to help the reader understand.

In addition to learning some interesting contemporary mathematics, the independent work gives the student training in the art of mathematical exposition. Students will **learn how to talk about and write mathematics in a way that is accessible, clear and interesting to others**. Communicating mathematics clearly is an essential tool for collaborative problem solving with a quantitative component. The junior year is an ideal time to learn and practice these skills, after the student has mastered the basics of classical analysis, algebra and geometry.

Junior Seminars and Junior Papers

Junior Seminars provide the experience of learning a currently active mathematical topic under the guidance of an experienced research mathematician who selects and prioritizes the background reading and works with individual students to help them develop their expository skills. Students read and digest and present the new ideas to each other, with assistance from the seminar leader. Working from source materials written by experts for other experts, students learn how a group of mathematicians working together on an unfamiliar problem masters the basics and begins to explore. A Junior Paper project is similar, but here the student works one-on-one with a research mathematician.

Both the junior seminars and the junior paper projects teach students how to learn mathematics by reading research books and papers and discussing them with an expert in the field. Both require the student to demonstrate their mathematical understanding by writing a paper at the end. The junior seminar emphasizes peer collaboration where students prepare, present and actively listen to mathematical talks, whereas the junior paper projects emphasize individual collaboration between a student and an expert.

The Senior Thesis

The primary goal of the senior thesis is to demonstrate the student's substantial progress toward acquiring state-of-the-art knowledge of a significant area in contemporary mathematics or its applications. To accomplish this, the student must make use of the general mathematical knowledge and logical reasoning skills acquired in the required course work for that major as well as the written and oral expository skills they have been developing in the junior independent work. The end result should be a written presentation and an oral presentation that showcases the student's accomplishments across four years of mathematical work.

The thesis topic can be pure or applied. The primary adviser can be a member of any department of the University, but the thesis should have a substantial mathematical component, and it should connect with and build on the mathematical knowledge acquired in the major courses. If the main topic of the thesis is outside mathematics, the departmental senior adviser should be consulted early on to ensure that the requirement of a substantial mathematical component will be adequately met.

Most students will demonstrate their progress by providing a clear, mathematically correct survey of the research area in which they have chosen to work. This survey should provide the background and motivation for work in this field, a discussion of the main questions of this area as well as standard methods and a summary of the current state of the subject. The thesis should display the student's knowledge and understanding of the relevant literature, laying out important connections and major contributions of others, correctly attributed. A typical thesis will include the student's own investigations into the questions of the field. In some cases, the thesis will include new results, suitable for publication, though this is never required.

There are three main types of senior thesis, although in practice, most theses are a combination of the three types.

1. **Developing a publishable new result:** For such a thesis it is appropriate to focus on a narrower topic after an initial broader survey of the field. The thesis should begin with an introduction aimed at the student's peers, followed by a thorough review of the relevant research literature. The thesis should explain how the new results compare with the state of the art. The new results should be presented in a form that is suitable for a published research paper. A new result may take various forms such as: a new theorem or a new proof of a previous established result, working out some substantially new examples, writing numerical simulations or computer programs and analyzing the results, or developing new insights into a question using mathematical tools or providing data analysis.
2. **Exploring connections between mathematics and other disciplines:** Such a thesis generally has two advisers, a primary adviser in a department outside of mathematics, and a secondary adviser in math. The thesis should start with a review of the relevant research literature, making sure that the treatment is appropriate for both mathematicians and for experts in the area of application. The thesis should continue with an explanation of how the new results compare with the state of the art, followed by a presentation of the results. Since the focus is on applications of mathematics, the standards in the field of intended application, as communicated by the primary thesis adviser in that field, should be the main guidance. It may also be appropriate to have a separate chapter explaining more of the mathematical background of the results and perhaps proving some of the theorems. The math department adviser will provide guidance on this question.
3. **Surveying a substantial area of contemporary mathematics:** Such a thesis should start with a review of the relevant literature. Next the thesis should explain how the new write-up compares with other books, survey papers and relevant lectures available online, and this review should be followed by the student's synthesis of the literature. The level of the presentation may vary. It could be a survey of an active research area, at the level of a chapter in an advanced textbook. It could be like a chapter in an introductory textbook, where the presentation has a distinct viewpoint, applications or worked-out examples that are substantially different from those found in published books and lectures. It might be in the form of an essay aimed at high school students or at a general readership, provided it has a viewpoint that is substantially different from published books and lectures. For such a thesis, there should be a

chapter that explains how the topic and the presentation connects with the mathematical knowledge acquired from the coursework in the major.

It is important to realize that it is not usually clear at the beginning which type of thesis a student will end up writing. To get started, we recommend that the student spend the first semester getting familiar with the topic, before deciding on the main plan for the thesis. The beginning of the second semester is a good time to take stock of the pace and progress of the work accomplished in the first semester in order to set realistic goals for the final project.

Other forms besides the three described here are possible, but in that case, it is essential to discuss the plan with the thesis adviser and the senior adviser early on to ensure that the expectations are clearly understood by everyone involved.

The department does *not* expect original mathematical research in junior and senior independent work. Many areas of mathematics have been under development for centuries, and the tight deadlines of the academic calendar are very restrictive. The undergraduate years should be used mostly for learning broadly and deeply. If a senior thesis student is able to approach a frontier of research and manages to prove something original in just a few months of work, this is an exciting achievement, but it cannot reasonably be the expected standard for a senior thesis or junior paper.

Evaluation

The various types of independent work in mathematics all have common goals which form the basis for evaluating a student's work.

- Develop and demonstrate **a working knowledge of a significant area of mathematics, showing substantial familiarity with the relevant literature**. The student reads, discusses and digests a broad selection of research books and papers in order to deliver a clear survey of the topic, in a paper or in a talk, where important definitions and theorems are clearly explained and properly acknowledged.
- Develop and demonstrate **an original point of view or individual mathematical style.** The student synthesizes the new topic and incorporates it into a general body of mathematical knowledge acquired through the student's prior coursework and mathematical experience. This may take the form of new mathematical results or insights or an original point of view through which the new research topic or its applications are presented.
- Demonstrate mastery of **rigorous logical thinking**. The arguments must be complete and correct, presented at a level of detail appropriate for the intended audience. For a typical junior paper, the intended audience is the student's peers, and so the detail expected is similar to that found in mathematical textbooks. If a thesis presents new results suitable for publication, then the presentation should have an introductory part aimed at the student's peers, and then a write-up of the new results aimed at the community of fellow experts in the field, with a level of exposition similar to that found in research articles.
- Demonstrate mastery of **clear mathematical exposition**, in a manner that allows and invites dialog with other scholars in the intended audience. Whether in a talk or a written paper, the presentation should place results in context within the general area of research, explaining the general problem under investigation, the methods used, and the results obtained, organized in a logical and accessible manner. The main ideas of the paper should be readily apparent. Previously known definitions and results should be correctly stated and properly acknowledged. Details should be correct, with well-chosen notation established and used

consistently. A conclusion containing a discussion of any general insights or open questions raised by the student's investigations is encouraged.

The grades for the independent work are set by the department based on recommendations from the main advisers and second readers. For junior independent work the seminar leader or JP adviser (with the second reader, if applicable) makes the grade recommendation to the department. For the senior thesis, the thesis adviser and second reader jointly make a grade recommendation. In the case of the senior thesis, two grades are assigned, one for the quality of the presentation and defense, and one for the thesis itself.

A paper or thesis earning an A should satisfy all four of the goals listed above. The paper covers a large portion of the chosen area. The student has mastered the material so well that their own voice comes through; the paper does not include large stretches whose organization simply follows one of the sources. There are no errors in the proofs. The exposition is clear and convincing.

A paper or thesis with a minor deficiency in one or two areas will earn a grade of A-, whereas a major deficiency in any single category will result in a grade of B+ or lower. Grades of B or B- are typically awarded to theses that are lacking in a significant way in one area and more minor ways in additional areas. Major deficiencies in two or more areas result in a grade of C.

For the most part, the difference between a major and a minor deficiency is a question of degree, that is, of how pervasive the problem is throughout the thesis. Broadly speaking, there are four main types of problem to watch out for:

- **Insufficient advanced mathematical content:** The thesis should build on and display well-developed connections to upper division course work in the major, making substantial use of ideas from analysis or topology/geometry or algebra or discrete mathematics at the 300-level or higher. A thesis focused entirely on the mathematics found in a 200-level course would be deficient, as would an applied math thesis that made no use of advanced mathematical techniques or concepts.
- **Insufficient engagement with the research literature:** The thesis should survey multiple advanced sources to put the mathematics problem it addresses into its proper context in a larger research area. The thesis should highlight the main techniques/questions of that research area and showcase how the various sources fit into an ongoing scholarly discussion amongst multiple contributors. A thesis that relies almost entirely on a single source, following a chapter of a book or a paper very closely with only minor changes, is incomplete. A thesis that merely reports on individual contributions from the literature sequentially, without considering their natural mutual implications or their consequences for the problem under investigation, would also be incomplete, but less so.
- **Insufficient attention to rigor:** Definitions or statements or proofs containing mathematical errors are always a problem. Depending on the how pervasive this issue is, this can be a major or a minor problem. In extreme cases, these kinds of errors can reveal serious gaps or fundamental errors in the student's understanding of the question being investigated and entirely nullify the results a student is reporting on. Early drafts written with careful attention to rigor

and clarity are an important tool to help the student catch serious errors in understanding early enough to correct them.

- **Insufficient attention to exposition:** Missing definitions, inconsistent notation, incomplete proofs or ideas whose provenance is unclear create obstacles for the reader to evaluate and respond to the mathematical discussion in the thesis, and in extreme cases entirely derail the scholarly discussion the student is attempting to join.

Identifying and correcting these deficiencies is a major part of the advising process. Regular advising meetings and early written drafts provided in time to receive feedback and revise accordingly is the best way to ensure that the final product meets the expected standard in all four categories. It is often difficult to suspend the hunt for new results and pause to write up preliminary progress carefully, but these pauses often produce new insights and they ensure that the final write-up is of higher quality.

For students whose primary senior thesis adviser is outside the math department, the further the work is from a mathematical topic, the more important it is to meet with an adviser in math (either the second reader or the senior adviser or the director of undergraduate studies) to ensure that the requirement of a substantial mathematical component is adequately met. For example, if a student builds a working time machine as a senior thesis project, the submitted thesis must nonetheless contain a detailed mathematical discussion of the underlying theory of time machines in order to earn a grade of A.

Some suggestions to students for writing a good mathematics paper

Make sure to motivate the subject matter of the paper by briefly describing the history of the topic and some specific reasons why the topic is important and interesting. The introduction should be a teaser that attracts the reader into the world of the paper.

State definitions and theorems clearly. Good grammar and clean suggestive notation and terminology are essential. Use the least amount of notation and terminology necessary to state the main theorem(s). Make sure to give examples throughout to help comprehension. For more detailed, technical, or lengthy arguments, try to give the global overall picture first so that the reader is not as afraid to delve into the details. Mathematicians frequently judge arguments by their "elegance". Complete arguments are also appreciated.

Some suggestions for giving a good mathematics lecture

Make eye contact with members of the audience regularly. Too often the speaker is talking only to the blackboard, or only to one person. Everyone should feel part of the experience.

Project the voice and enunciate. Speaking in a normal voice will sound like mumbling during a lecture. Make sure to project the voice so even those sitting in the back can hear it clearly.

Write legibly and large enough. Use blackboards wisely and in an organized manner, typically from top to bottom and then left to right. Draw lines on the blackboard to separate writing from different trains of thought.

It is extremely important to state the definitions and theorems in full. Saying something orally doesn't mean that it doesn’t need to be written down too. Here is a rule of thumb for what to write on the board: if audience members were to take notes by simply copying down what is written on the

blackboard, without listening to a word that is said, they should still be able to understand and appreciate the lecture if they look at their notes years from now.

A more advanced skill: don't be shy to repeat and re-repeat statements to drive home important points.

Feel free to underline or put boxes around important formulas or concepts. Colored chalk can also help in creating appropriate emphasis. Feel free to be creative.

Learn to be able to continue to talk while writing, erasing, and moving blackboards. This helps to avoid dead time during the lecture.

However, when presenting more difficult points, or every once in a while, be sure to make an appropriate pause. This will give the audience members time to digest what has been said or to ask questions. Ask, "Are they any questions?" every so often.

A good lecture should contain at least one theorem, one example, and one joke!

Timeline and Important Deadlines

Junior Seminars and Junior Papers

Second week of semester: Attend organizational meeting for junior seminars. Confirm participation in a junior seminar with the undergraduate program office. Students not participating in a junior seminar should notify both the undergraduate office and the faculty junior adviser that they are writing a junior paper; turn in completed form listing the JP topic and JP adviser, signed by the JP adviser.

Each week: Junior seminars meet weekly, typically for two hours. For a JP, the student and adviser have a weekly meeting where the student reports on what they have been reading and thinking about.

Tenth week of semester (two weeks before reading period): It is recommended that a draft of the JP be given to the reader(s).

University deadline for junior independent work: **All JPs and seminar papers are due.** Please submit a .pdf of the paper to the adviser and to the undergraduate program office.

Senior Theses

Each week: The student and adviser have a weekly meeting where the student reports on what they have been reading and thinking about.

Fourth week of fall semester: First thesis report due, including the name of thesis adviser and preliminary description of thesis topic, signed by the thesis adviser. The undergraduate program office will email the report form.

Fourth week of spring semester: Thesis progress report due, including a request for the name of a second reader.

Tenth week of spring semester (two weeks before reading period): It is recommended that a draft of the thesis be given to the two readers.

Early in Reading Period: **Thesis due** (see Academic Calendar for exact date). Typically, a copy of the thesis must be delivered to the undergraduate program office. A .pdf file must also be emailed to the undergraduate office and uploaded to Thesis Central by the student. Copies should be given to the two readers. There are no special departmental binding requirements for senior thesis, beyond those required by the University.

Reading period: Confirm scheduling of departmental exam and any special requests (media needs, etc.)

Senior departmental exams: usually just prior to the start of exam period.

Extensions: Extensions to the final deadlines are rare and granted only for exceptional circumstances. Requests for extensions must be discussed with the junior seminar leader or JP or thesis adviser and with the faculty junior and senior advisers in the mathematics department. Ultimately, however, extensions are granted only by the appropriate college deans.

Process and Advising

Junior Independent Work

All mathematics majors are expected to participate in a junior seminar during at least one semester of their junior year. In the other semester, they can either take a second junior seminar or write a junior paper.

Junior Seminars

Junior seminar topics for each semester will be announced at the end of the previous semester. An organizational meeting will be held at the start of each semester, at which time students formally choose a seminar in which to participate.

A junior seminar typically meets about once a week for about two hours. After introductory lectures by the seminar leader in the initial couple of meetings, the lectures are turned over to the student participants. Typically, one junior will speak in each hour.

<u>Presentations</u>

Each seminar generally consists of two 50-minute lectures by a pair of students. It is recommended that the two students plan their lectures together and give practice talks to each other to ensure that the lectures are clear and accessible to the rest of the class. A day or two before the presentation, the students meet with the seminar leader to share notes and to do (at least a partial) practice run-through on the blackboard, to get additional feedback and suggestions for fine-tuning.

A student would generally participate in two such presentations during the term, with different partners.

<u>Writing</u>

About a month before the university deadline for submission of junior independent work (after the Fall reading period in January, or at the start of the Spring reading period in May), the seminar leader will suggest a list of possible topics for a final short paper of 5-8 pages. Each participant will work independently on this final paper, on a topic of the participant's choosing. Papers should be of interest

and accessible to the participants of the seminar, requiring no further background beyond the seminar material.

A week or two before the due date, the participants give a mostly polished draft to the seminar leader for comments. The final product is due by the university deadline for junior independent work.

Junior Papers

During one term of their junior year, students may choose to write a junior paper (JP) in lieu of a junior seminar. If they do a JP, students should arrange the topic with an adviser and make an initial plan for the project. By the end of the second week of the term, they should give the undergraduate office the name of the adviser and a tentative title. The student and adviser should meet weekly to discuss the student's work and to look for new mathematical directions in which the project can grow. JPs are due by the university deadline for junior independent work (after the Fall reading period in January, or at the start of the Spring reading period in May).

Junior papers tend to be more substantial in both length and content than papers written during junior seminars. A typical length is 15 pages or more. However, there are no formal length requirements.

A JP may consist of an exposition of a novel approach to, or perspective on, a particular area or problem in pure or applied mathematics. It may also present a sophisticated connection between different areas of mathematics that at first seemed separate. In very rare cases, the paper or thesis may consist of new theorems, conjectures, computations, or applications that were previously unknown. In all cases, the emphasis is on the ability to delve deeply into a mathematical area and to communicate mathematical ideas in this area with rigor, motivation, clarity, and logical reasoning.

A JP is expected to be at least as much work as a regular course. For a JP, the student and adviser have a weekly meeting where the student reports on what they have been reading and thinking about. In the early weeks, the adviser will help the student develop ideas, correct the ideas (if necessary), and recommend further reading as appropriate. Later, the adviser will read drafts of the paper and will make comments to improve the student's writing.

Sometimes problems arise, where a student has difficulty finding a JP adviser or has not met with their adviser for a long time. Students are urged to communicate with the math department if such problems arise. The Junior Adviser, and the undergraduate math administrator, are always happy to talk with students and to do their best to help with problems. The earlier the problems are addressed, the better. When staff members suspect there is a problem and write to the student, the student is warmly urged to respond immediately, so that the department can help them.

Junior papers are evaluated primarily by the adviser; a second reader is not required for junior independent work. However, if the primary adviser is outside the mathematics department (which is allowed, supported, and indeed occurs frequently), a second reader from the mathematics department must be arranged. This is to help ensure uniformity in the evaluation process and to ensure that the requirement of a substantial mathematical component is adequately met.

Senior Theses

Senior theses are expected to be more in-depth, more original in exposition and content, and more substantial than junior papers. However, there are no formal length requirements. Some of the best senior theses in mathematics over the years have been under 10 pages, while others have been over 100 pages. About 20-40 pages when single-spaced seems typical.

Like a JP, a senior thesis may consist of an exposition of a novel approach to, or perspective on, a particular area or problem in pure or applied mathematics. It may also present a sophisticated connection between different areas of mathematics that at first seemed separate. In rarer cases, the paper or thesis may consist of new theorems, conjectures, computations, or applications that were previously unknown. In all cases, the emphasis is on the ability to delve deeply into a mathematical area and to communicate mathematical ideas in this area with rigor, motivation, clarity, and logical reasoning.

A senior thesis is expected to be the main part of a student's workload during the senior year. The student and adviser have a weekly meeting where the student reports on what they have been reading and thinking about. In the early weeks, the adviser will help the student develop ideas, correct the ideas (if necessary), and recommend further reading as appropriate. Later, the adviser will read drafts of the paper or thesis and will make comments to improve the student's writing.

As with the JP, problems sometimes arise, where a student has difficulty finding an adviser or has not met with their adviser for a long time. Students are urged to communicate with the math department if such problems arise. When staff members suspect there is a problem and write to the student, the student is warmly urged to respond immediately, so that the department can help them.

The senior thesis is due by the deadline set by the University, usually at the start of the spring reading period. The thesis is evaluated by both the adviser and a second reader. The second reader should be arranged at least by early in the Spring semester.

The thesis is "defended" to both the adviser and the second reader during a thesis defense that occurs during a period of a few days set by the University, usually just after Dean's Date. The defense consists of a short (about 25 minute) presentation by the student (see the exposition suggestions below), followed by questions from the readers pertaining to the content and background of the thesis. Two grades are given: one for the quality of the presentation and defense (to emphasize the importance of mathematical communication), and one for the thesis itself.

The primary adviser for the senior thesis can be a university faculty member outside the mathematics department (which is allowed, supported, and indeed occurs frequently). In this case, the second reader must be arranged within the mathematics department. Again, this is to help ensure uniformity in the evaluation process and to ensure that the requirement of a substantial mathematical component is adequately met.

Grades on the junior independent work and senior thesis, together with grades in mathematics courses, are taken into account when determining departmental honors.

Finding and Working with an adviser

Some of the best advice for finding an adviser comes from those who have recently done so. The following advice is taken from the Princeton University Math Club's *Guide for Math Students* and is drawn from the experience of recent math majors.

"Starting a thesis or JP requires two major steps: choosing an adviser and choosing a topic. In the overwhelming majority of cases, the former comes first. Choosing your adviser carefully is important. Your adviser's style and the compatibility between the two of you will deeply influence the quality of your experience ... you should consult older students and the faculty academic advisers to figure out which professors might be a good fit for you in terms of research interests and advising style. As always, you will have to be proactive to ensure your experience is all that it can be; ask your peers many questions: about the frequency and content of meetings, the expectations for an undergraduate project (too low? too high?), the level of preparation expected, and so on.

Once you have an adviser, you will still need to find an effective way to work together. Sometimes, this will come naturally; that's especially likely if your adviser often takes undergraduate students. Be that as it may, figure out how often meeting with your adviser is productive; once a week is standard, but some professors prefer biweekly meetings. Even if you have nothing to report, meeting with your adviser helps both of you stay in touch with the project and is an integral part of the research experience. You will also want to prepare for your meetings so as to get the most out of them. While meetings will be your primary interaction with your adviser, emails and other day-to-day interactions can be nearly as important. These generally take the form of questions and, here again, you will need to figure out how to make them work best for you. Experience is, for better or for worse, the only real way to learn how to do this. Finally, whoever your adviser is, you will benefit from making friends with their graduate students and postdocs (short for "postdoctoral fellows," researchers who have recently obtained Ph.D.s); they can serve as secondary advisers who can help you on a day-to-day basis—and share their experiences with early career research.''

This and other potentially useful advice from recent math majors can be found at

http://blogs.princeton.edu/mathclub/guide/research/

The primary adviser for a JP or senior thesis can be a university faculty member outside the mathematics department (which is allowed, supported, and indeed occurs frequently). In this case, the second reader should be arranged within the mathematics department. Again, this is to help ensure uniformity in the evaluation process.

Funding

The main reason math majors apply for funding is to do mathematical reading and research over the summer. The summer program is primarily to support work with a faculty member of the Princeton Math Department. In some cases, we also support participation in a summer research program elsewhere in the US, typically Research Experiences for Undergraduates (REUs) at other schools. Students must have been enrolled (i.e., not on leave) during the academic year before the summer for which they are applying.

Reading/Research at Princeton. The goal of this program is to provide math majors with the opportunity for reading and study in some areas of mathematics that are not part of the regular program of courses offered by the Mathematics Department. Ideally, this would be an opportunity for each student to learn not just a body of mathematics but also to be able to engage in some independent investigation. We expect that supported students will devote at least 20-25 hours per week to reading and research. The stipend in Summer 2023 was $4800.

Students interested in applying must find a faculty member in the Princeton Mathematics Department who agrees to serve as the student's adviser for the period of the program. Part of the application will be a proposal, a substantial description of the topic that the student will pursue. Before the program is approved, the adviser and the student will outline the program of reading and study. The adviser will meet regularly with the student during the summer. The organization and conduct of each student's program will be up to the individual adviser; implementation could vary considerably depending on the topic and goal that the student and adviser agree upon.

The summer research program runs for twelve weeks. Students will be required to meet with their advisers for at least eight weeks during the summer period. The eight weeks do not have to be consecutive. We recommend that the meetings be in person, but they can be by Zoom if both parties agree.

At the end of the summer, the student will submit a report detailing their research activities to the Undergraduate Administrator.

Participation in other programs.

Math majors who are requesting funding to attend a summer REU elsewhere in the US will need to submit the following: a description of the REU program, including any links to the program; an outline of the proposed activity; and information on the funds they are requesting. Financial information on each REU should also be included in the description of the program. A letter of recommendation from a faculty adviser is *not* required in this instance.

For most REUs, students do not have to apply for funding—indeed, the REU pays the students. However, some REUs cannot pay everyone. For example, most REUs cannot pay a student who is not a US citizen. That is where the math department's funding comes in.

If math department funding is awarded, the student will be required to submit the letter of acceptance into the REU before funds are paid out. We do not typically provide funding if the REU is also providing support.

The student can apply for both REU funding and a research project with a Princeton faculty member, as described above. They should indicate their preferred project, and they should let the potential faculty adviser know that they are applying for both.

How to apply. An applicant must be a math major who will be a senior at Princeton University in the Fall semester after the summer program, or a current sophomore at Princeton who will declare as a math major. An applicant should upload an outline of the proposed activities to the SAFE portal at http://www.princeton.edu/studentfunding/. To search for our funding opportunity, the activity type is

"independent project"; the location is "on campus" or "off campus (domestic)"; the duration is "8 or more weeks"; and the time of year is "summer break". The student must also indicate the name of the faculty adviser (unless the application is for REU funding). The adviser will be asked to upload a memo or letter stating that they approve the program of reading and study and that they agree to serve as the student's adviser. The adviser should look for the place to enter data as a "recommender".

All completed proposals will be reviewed by the Chair and Associate Chair. In the event that the department receives more proposals than it can fund, preference will be given to current juniors over current sophomores.

The application deadline is typically in mid-February. It is announced each year, three to four weeks in advance.

Funding to speak at conferences. When math majors are giving a talk at a conference on mathematical work they have done, either as a lecture or in a poster session, the math department is able to reimburse some of their expenses for travel, lodging, and application fees. This is only for math majors—not, for example, for first-year students or for majors in another field. There is a cap on the expenses for each conference. In Spring 2023, the cap was $750.

Style, Structure, and Format

The math department does not impose style guidelines of its own on the seminar papers, junior papers, or senior theses. In broad terms, students are asked to follow the standard of writing of the mathematical books and articles they have read over the years. For specific guidance, students are asked to consult the bound copies of senior theses stored (before 2020) in Fine Hall Library, or the .pdf copies of recent theses on Thesis Central at https://thesis-central.princeton.edu/.

LaTeX is the international standard for the typesetting of mathematical writing. Students are strongly encouraged to write their theses, JPs, and other mathematical papers in LaTeX.

For the senior thesis, the University imposes a few style guidelines, such as a title page and a page for the advisers' signatures. A LaTeX template for senior theses that meets these requirements is available at

- puthesis-UG.cls - the actual style file
- puthesis.sample.tex - first sample LaTeX file that shows how to use puthesis-UG.cls

Please note that the file puthesis-UG.cls must be saved in the same directory as the thesis TeX file. Further, puthesis-UG.cls includes an Honor Code declaration; by using this template, a student declares that the paper represents their own work in accordance with University regulations.

Resources for independent work

1. The thesis or JP adviser.

The best resource for learning about mathematical research is someone who does it as a profession. Do not be afraid to ask lots of questions!

2. The library.

Lewis Library contains an exceptional collection of mathematical books and journals. Browsing in the shelves often leads to unexpected connections. Also, many journals and books are available online (sometimes only online) if one logs on to the Princeton Library website with their University ID.

The library contains bound copies of all senior theses from before 2020. These provide examples of what to do (and not to do!) when writing mathematics.

3. The Internet.

There are many helpful online resources: MathSciNet (which contains reviews of most published papers in mathematics), electronic versions of journals and books, preprint servers, and web pages of mathematicians. When students need assistance with any of these, the Lewis Library staff can help. As always on the Internet, use judgment to decide whether a site is safe and whether its contents are correct.

4. Other students.

Talking to other students about their experience doing mathematical research can be both helpful and stimulating.

There is a reason mathematicians spend so much time at conferences: explaining one's work and hearing others explain theirs is a big part of doing mathematics. It can help clarify ideas and provide new insight.

The Princeton Undergraduate Mathematics Club has compiled advice from the experiences of many students:

http://blogs.princeton.edu/mathclub/guide/research/

5. The junior and senior departmental advisers.

Talk to them about any problems or difficulties, no matter how small, before they become big.

6. The undergraduate program office.

The source for all questions about procedures and deadlines.

7. The Writing Center.

The Writing Center, located in New South, offers an array of options for help writing JPs and senior theses:

http://www.princeton.edu/writing/center/

Contacts

János Kollár

Associate Chair, Director of Undergraduate Studies
kollar@math.princeton.edu

Jennifer Johnson

Associate Director of Undergraduate Studies
jmjohnso@math.princeton.edu

Alexandru Ionescu

Senior Adviser (2023-2024)
aionescu@math.princeton.edu

Mark McConnell

Junior Adviser
markwm@princeton.edu

Michelle Matel

Undergraduate Administrator
mmatel@princeton.edu