

## Final for Math 325

This is a take home exam, due on May 20, 2007. Some of the problems are from Munkres, Topology, Second Edition. You are encouraged to use your notes and the textbook.

1. page 158/ problem 12
2. page 223/7 (note that this problem has two parts,  $a$  and  $b$ )
3. 367/10
4. 457/1
5. 457/3
6. 457/4
7. Let  $G$  denote the group given by the following generators and relations:

$$G = \langle a, b \mid a^3 = b^2, a^3 = (a^{-1}b)^5 \rangle$$

Show that  $G$  is not the trivial group. (**Hint:** Let  $S_n$  denote the permutation group on  $n$  elements. Try to find a non-trivial group-homomorphism from  $G$  to  $S_5$ )

**8.** In this problem we will use complex coordinates  $z$  and  $w$ . We define the three-dimensional sphere by

$$S^3 = \{(w, z) \in \mathbf{C} \times \mathbf{C} \mid |w|^2 + |z|^2 = 1\}$$

**8/a.** Show that the subset of  $S^3$  where

$$|w| \leq \frac{1}{2}$$

is homeomorphic to  $S^1 \times D^2$ , where  $D^2$  denotes the two dimensional disk.

**8/b.** Let  $\Gamma$  denote the subset of  $S^3$  given by the equation:

$$w^3 + z^5 = 0$$

Show that  $\Gamma$  is homeomorphic to a circle.

**8/c.** Compute the fundamental group of  $S^3 - \Gamma$ .