

## Princeton Discrete Math Seminar

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Thursday, December 6th

Department of Mathematics

2:15-3:15pm

Fine Hall, Room 224

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### Critical triangle-free graphs with lots of edges

Wesley Pegden

Rutgers University

How close are the triangle-free graphs to the bipartite graphs? The different ways of asking this question give different answers. For example: on the one hand, triangle-free graphs can have arbitrarily large chromatic number; on the other, natural constructions to demonstrate this fact are typically very sparse. One type of question with this theme is the recently solved question of Erdős and Simonovits, which asks about triangle-free graphs with large chromatic number and large minimum degree.

We consider a different question with the same motivation: are there  $k$ -chromatic-critical triangle-free graphs with lots of edges for arbitrarily large  $k$ ? We'll talk about a construction that shows that the answer is yes. We'll also show how one can nonconstructively extend the result to graphs without pentagons as well. Results about the density of general critical graphs typically leave wide gaps between what we know as upper and lower bounds; the case of triangle-free graphs seems unusual in this context, since our construction actually shows the exact maximum asymptotic density.