

MATH 306 — HOMEWORK 9

Due in class on Tuesday, April 15th

1. What is the maximum possible number of edges in a simple planar bipartite graph with $n \geq 3$ vertices? Deduce that $K_{3,3}$ is not planar.
2. Let G be non-null, simple and planar, with no vertex of degree ≤ 4 . Is it true that G must have two vertices of degree 5 which are joined by a path of length < 100 ? Is it true that if $|V(G)| > 100,000$ then G has ≥ 13 vertices of degree 5?
3. G is **outerplanar** if it can be drawn in the plane so that every vertex is incident with the infinite region. Show that a graph G is outerplanar if and only if G has no K_4 or $K_{2,3}$ minor. [Hint: add a vertex appropriately and use Kuratowski's theorem.] Deduce that if G is simple and outerplanar and $V(G) \neq \emptyset$ then G has a vertex of degree ≤ 2 .
4. Let G be 2-connected and series-parallel. Show that G has a Hamilton cycle if and only if G is outerplanar.