

MATH 306 — HOMEWORK 8

Due in class on Tuesday, April 8th

1. A graph G is a *cactus graph* if every two distinct cycles in G have at most one vertex in common. Let B_3 be a graph with 2 vertices and three parallel edges.
 - (i) Show that a graph is a cactus graph if and only if it does not contain a minor isomorphic to B_3 .
 - (ii) Show that every loopless 2-connected cactus graph is a cycle.
 - (iii) Show that a graph G is a cactus graph if and only if G can be constructed from the null-graph by repeatedly applying the following operations: adding a vertex of degree ≤ 1 ; or adding a loop; or subdividing an edge.
 - (iv) Show that if G is a non-null loopless cactus graph then $|E(G)| \leq 2|V(G)| - 2$.
2. How many simple 2-connected graphs G are there with $|V(G)| = 100$ such that
 - (i) G has no C_4 minor? (C_k is the cycle of length k .)
 - (ii) G has no C_5 minor?
 - (iii) G has no $K_4 - e$ minor? ($K_4 - e$ is the graph obtained from K_4 by deleting one edge.)

[Note: These are three different questions, do not work out which G has **none** of these three minors.]