MATH 306 — HOMEWORK 7

Due in class on Tuesday, April 1st

1. Show that every 3-edge-connected cubic graph has a perfect matching. (Hint: if $X \subseteq V(G)$, $|\delta(X)| \leq 3|X|$.) What about 2-edge-connected cubic graphs?
2. Let G be a graph and $Z \subseteq V(G)$. Show that the following are equivalent:
   (i) G has a matching covering Z
   (ii) for every $X \subseteq V(G)$ there are at most $|X|$ odd components C of $G \backslash X$ such that $V(C) \subseteq Z$.

[Hint: add extra vertices appropriately.]

3. Show that the vertices of any loopless graph G can be covered by at most $\alpha(G)$ vertex disjoint subgraphs of G each isomorphic to a cycle or a complete graph on at most 2 vertices. ($\alpha(G)$ denotes the size of a maximum independent set in G.)

[Hint: Find a subgraph H of G isomorphic to a cycle or a complete graph on at most 2 vertices that contains a vertex not adjacent to any vertex in $V(G) - V(H)$. Then apply induction on $\alpha(G)$.]

4. Show that for any integer $n > 0$ there exist an integer N so that any sequence $x_1, x_2, \ldots, x_N$ of N distinct real numbers contains an inreasing subsequence of length n or a decreasing subsequence of length n. (An increasing subsequence is a sequence $x_{i_1}, x_{i_2}, \ldots, x_{i_n}$ such that $x_{i_1} < x_{i_2} < \ldots < x_{i_n}$ and $1 \leq i_1 < i_2 < \ldots < i_n \leq N$.)