MATH 306 — HOMEWORK 4

Due in class on Tuesday, March 4th

1. Let G be a bipartite graph, with bipartition (A, B). Show that the following are equivalent:
   (i) there is a matching M in G so that every vertex in A is an end of some member of M
   (ii) for every $X \subseteq A$ there are at least $|X|$ vertices in B with a neighbour in X.
2. Let G be bipartite, so that every vertex has the same degree k say. Show that $E(G)$ can be partitioned into perfect matchings. (Hint: first show that if $k > 0$ there is a perfect matching.)
3. Show that a graph G is bipartite if and only if for every subgraph H of G, there is a subset $X \subseteq V(H)$ with $|X| \geq \frac{1}{2}|V(H)|$ so that no two members of X are adjacent in H.
4. Let G be a loopless graph in which every vertex has degree ≥ 1. Let X be the largest matching in G, and let Y be the smallest set of edges of G so that every vertex of G is incident with ≥ 1 edge in Y. Show that $|X| + |Y| = |V(G)|$.