

MATH 306 — HOMEWORK 3

Due in class on Tuesday, February 26th

1. If G is connected and $v \in V(G)$, v is **deletable** if $G \setminus v$ is also connected. Up to isomorphism, how many connected simple graphs G are there with $|V(G)| = 100$ and with at most two deletable vertices?
2. Let H be a complete graph with seven vertices A, B, \dots, G . For each edge e with ends u, v say, let $w(e)$ be as given in the following table (row u , column v).

	A	B	C	D	E	F	G
A	0	20	27	51	40	25	46
B	20	0	22	44	30	34	52
C	27	22	0	42	15	10	53
D	51	44	42	0	50	37	57
E	40	30	15	50	0	17	55
F	25	34	10	37	17	0	54
G	46	52	53	57	55	54	0

- a) What is the smallest k so that some spanning tree T of H has

$$\sum_{e \in E(T)} w(e) = k?$$

- b) For every $v \in V(H)$ find $\text{dist}_H(A, v)$.

3. Let T_1, T_2 be two spanning trees of a graph G , and let $e_1 \in E(T_1) \setminus E(T_2)$. Show there is an edge $e_2 \in E(T_2) \setminus E(T_1)$ such that e_2 belongs to the fundamental cycle of e_1 with respect to T_2 AND e_1 belongs to the fundamental cycle of e_2 with respect to T_1 .
4. Let G be a connected graph and for each edge e let $w(e)$ be some real number. Let T be a spanning tree of G such that for every edge $f \in E(G) - E(T)$, $w(f) \geq w(e)$ for every edge e of the fundamental cycle of f . Show that T is a min-cost tree. (Hint: use question 3.)
5. For a given n find the maximum l so that there exist bipartite graphs H_1, H_2, \dots, H_n with $H_1 \cup H_2 \cup \dots \cup H_n = K_l$, where K_l denotes the complete graph on l vertices.