MATH 306, Spring 2008 — HOMEWORK 2

Due in class on Tuesday, February 19th

1. Let G be a graph with $|E(G)| = |V(G)| - 1$. Show that G is connected if and only if G has no cycles.
2. Let T be a tree, and let T_1, T_2 be connected subgraphs of T with $V(T_1 \cap T_2) \neq \emptyset$. Show that $T_1 \cup T_2$ and $T_1 \cap T_2$ are trees.
3. Let T be a tree, and let $T_1, \ldots, T_n$ be connected subgraphs of T so that $V(T_i \cap T_j) \neq \emptyset$ for all i, j with $1 \leq i < j \leq n$. Show that $V(T_1 \cap T_2 \cap \ldots \cap T_n) \neq \emptyset$. [Hint: Delete a leaf and use induction on $|V(T)|$.]
4. Show that if $E(G) \neq \emptyset$ and G has no leaves then G has a cycle. Deduce that if every vertex of G has even degree then there is a set $C_1, \ldots, C_k$ of cycles of G so that every edge of G is in exactly one of them.
5. Let G be a graph which has a closed walk of odd length. Is it true that G has a cycle of odd length?