

## MATH 306 — HOMEWORK 11

Due in class on Tuesday, April 29th

1.
  - (i) How many cubic graphs  $G$  are there (up to isomorphism) such that  $|V(G)| = 10$  and every cycle of  $G$  has length  $\geq 5$ ?
  - (ii) Let  $G_1$  be the line graph of  $K_5$ , let  $G_2$  be the complement of  $G_1$  and let  $G_3$  be the line graph of  $G_2$ . What is the chromatic number of  $G_3$ ?
2. Let  $C$  be a cycle of length  $2k + 1$  where  $k \geq 1$  is an integer. For each edge  $e$  of  $C$  let  $n(e) \geq 0$  be an integer. Let  $G$  be constructed from  $C$  by replacing each edge  $e$  by  $n(e)$  parallel edges. Show that

$$\chi'(G) = \max(\Delta(G), \lceil |E(G)|/k \rceil) .$$

[Hint: For  $\leq$ , choose a matching of  $G$  covering all vertices with degree  $\Delta(G)$  if possible and delete it.]

3. Let  $G$  be a connected loopless graph with  $|E(G)|$  even. Is it necessarily true that the edges in  $E(G)$  can be paired up so that the two edges in each pair have a common end?
4. Suppose  $G$  is loopless, and that every non-null simple minor of  $G$  has a vertex of degree  $\leq t$ . Prove that either  $G$  has  $K_{t+1}$  as a minor or  $\chi(G) \leq t$ .