

MATH 306 — HOMEWORK 10

Due in class on Tuesday, April 22th

1. Let G be a 2-connected loopless graph drawn in the plane. For each vertex v , define $S(v) = \frac{1}{2} - \frac{1}{\deg(v)}$. Show that for some region r , $\sum_{v \sim r} S(v) < 1$, where the sum is over all vertices v incident with r . [Hint: what is the average of $\sum_{v \sim r} S(v)$ over all regions r ?]
2. Let G be a 2-connected simple planar graph, in which every vertex has degree ≥ 5 . Is it necessarily true that G has two adjacent vertices u, v with $\deg(u) = 5$ and $\deg(v) \leq 6$?
3. Show that if every two odd cycles in a simple graph G have a vertex in common then $\chi(G) \leq 5$.
4. Does there exist a graph G with no K_3 subgraph and $\chi(G) = 100$?