

MATH 306 — HOMEWORK 1

For each of the following statements, decide if it is true or false, and either prove it or give a counterexample.

1. Every simple graph with ≥ 2 vertices has two vertices with the same degree.
2. There is a simple graph with 8 vertices total, with degrees 2,2,3,3,3,4,4,4.
3. If u, v, w are vertices of G , and there is an even length path from u to v and an even length path from v to w then there is an even length path from u to w .
4. If G is connected and has no path with length $> k$, then every two paths in G of length k have at least one vertex in common.
5. If u, v, w are vertices of G , and there is a cycle of G containing u and v , and a cycle containing v and w , then there is a cycle containing u and w .
6. If e, f, g are edges of G , and there is a cycle containing e, f and a cycle containing f and g , then there is a cycle containing e and g .