The shooting method and the analysis of the target map via the degree theory

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Abstract

We introduce and analysis the 'target map' for the shooting method. For a large class of elliptic systems as well as more general dynamic systems, we show that the target map is **onto** via the degree theory. The target map is onto implies that we can shoot to any desired position.

Applying our result to a motivating example, we obtain the existence of global positive solutions to the Hardy-Littlewood-Sobolev (also known as Lane-Emden) type system:

$$\begin{cases} (-\triangle)^k u = v^p, & \text{in} \quad \mathbb{R}^n, \\ (-\triangle)^k v = u^q, & \text{in} \quad \mathbb{R}^n, \end{cases}$$

in the critical and supercritical case $\frac{1}{p+1} + \frac{1}{q+1} \geq \frac{n-2k}{n+2k}$. On the other hand, the Lane-Emden conjecture state that when $k = 1$, the above system admits no positive solution in the subcritical case $\frac{1}{p+1} + \frac{1}{q+1} < \frac{n-2k}{n+2k}$. The Lane-emden conjecture has been proved for many cases and no counter-example has been found yet.

This new approach to the shooting method is simple and powerful. We expect many more applications of it.