Princeton Discrete Math Seminar

Thursday,April 3rd   Department of Mathematics

2:15-3:15pm   Fine Hall, Room 314

Points Surrounding the Origin

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For $d > 2$ and $n > d + 1$, let $P = \{p_1, \ldots, p_n\}$ be a set of points in $\mathbb{R}^d$ whose convex hull contains the origin O in its interior. We show that if $P \cup O$ is in general position, then there exists a d-tuple $Q = \{p_{i_1}, \ldots, p_{i_d}\} \subset P$ such that O is not contained in the convex hull of $Q \cup \{p\}$ for any $p \in P \setminus Q$. Generalizations of this property are also considered.

We also show that for disjoint, non-empty, finite point sets $A_1, \ldots, A_{d+1}$ in $\mathbb{R}^d$ in general position with respect to the origin, if the origin is contained in the convex hull of $A_i \cup A_j$ for all $1 \le i < j \le d + 1$, then there is a simplex S containing the origin such that $|S \cap A_i| = 1$ for every $1 \le i \le d + 1$. This is a generalization of Bárány's colored Carathéodory theorem, and dually, it gives a spherical version of Lovász' colored Helly theorem.

Joint work with Andreas Holmsen and Helge Tverberg.