NONCONVERGENCE EXAMPLES IN AVERAGING

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Abstract. Systems which combine fast and slow motions lead to complicated two scale equations and the averaging principle suggests to approximate the slow motion by averaging in fast variables. When the fast motion does not depend on the slow one this approximation usually works for all or almost all initial conditions but when the slow and fast motions depend on each other (fully coupled), as is usually the case, the averaging prescription cannot always be applied, and when it is valid then only in the sense of convergence in measure (or in average) with respect to initial conditions. A nonconvergence example for fixed initial conditions constructed for small perturbations of integrable Hamiltonian fast motions is due to Neishtadt and it is based on the well known phenomenon of resonances there. We construct nonconvergence examples in the discrete time averaging setup in a completely different situation where fast motions are expanding maps and Markov chains. In this case large deviations results provide an exponentially fast convergence in measure on initial conditions while for almost all fixed initial conditions there is no convergence at all. The proof for Neishtad's example requires only elementary ordinary differential equations tools but even for simplest expanding maps of the circle the proof of non convergence is not trivial and it relies on thermodynamic formalism and large deviations results. It seems that this situation is typical for chaotic fast motions but how to extend the proof to even a bit more general situation is not clear yet. The work is joint with Victor Bakhtin.