Speaker: Idris Assani, Dept of Mathematics, University of North Carolina at Chapel Hill,
Title: The Bilinear Hardy-Littlewood function for the tail

Abstract:

The bilinear Hardy-Littlewood maximal function is defined for f, g measurable functions as

$$M^*(f,g)(x) = \sup_t \frac{1}{2t} \int_{-t}^{t} f(x+s)g(x+2s)ds.$$

A simple application of Hölder's inequality shows that $M^*(f,g)(x)$ is almost everywhere finite if $f \in L^p$, $g \in L^q$ and $\frac{1}{p} + \frac{1}{q} \leq 1$. A. Calderón (1960) made a famous conjecture by stating that M^* is integrable as soon as f and g are in L^2. M. Lacey built on his work with C. Thiele on Carleson-Hunt celebrated theorem on the almost everywhere convergence of Fourier series to solve Calderón's conjecture. He showed that actually M^* maps $L^p \times L^q$ into L^r as long as $p, q \geq 1$, $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$ and $r > 2/3$. However nothing is actually known about the finiteness of $M^*(f,g)(x)$ when $3/2 \leq \frac{1}{p} + \frac{1}{q} \leq 2$. This question appeared also in the recent work of C. Demeter, T. Tao and C. Thiele. Transferred to the ergodic setting one looks at the equivalent problem for the Fürstenberg averages (introduced in 1977)

$$F_N(f,g)(x) = \frac{1}{2N+1} \sum_{n=-N}^{N} f(T^n x) g(T^{2n} x).$$

Using mainly ergodic theory tools we completely characterize the values $p, q \geq 1$ for which $R^*(f,g)(x) = \sup_{n \geq 1} \frac{f(T^n x) g(T^{2n} x)}{n}$ is a.e. finite. This is a joint work with Z. Buczolich.