Soliton collision for the nonintegrable gKdV equations

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Abstract

In this talk, we are concerned with the subcritical gKdV equations

$$\partial_t u + \partial_x(\partial_x^2 u + u^p) = 0 \quad t, x \in \mathbf{R}, \tag{1}$$

for $p = 2$, 3 and 4. We mainly focus on the nonintegrable case $p = 4$. Equation (1) is known to have special solutions of the type $u(t,x) = Q_{c_0}(x - x_0 - c_0 t)$, called solitons. The general problem is the following: one knows the existence of solutions of the equation which behave as $t \to -\infty$ like

$$u(t,x) = Q_{c_1}(x - x_1 - c_1 t) + Q_{c_2}(x - x_2 - c_2 t) + \eta(t,x), \tag{2}$$

where $c_1 > c_2$ and $\eta(t)$ is a dispersion term small in the energy space H^1 with respect to Q_{c_1}, Q_{c_2}. The two solitons Q_{c_1} and Q_{c_2} have collide at some time t_0. Can one understand the collision and determine what happens after the collision? In nonlinear analysis, except for some completely integrable equations, these questions are completely open.

We introduce a new framework to understand these problems for (1) in the case where $c_2 \ll c_1$ and $\|\eta(t)\|_{H^1} \ll \|Q_{c_2}\|_{H^1}$. First, this approach allows us to describe for all time solutions satisfying (2) for t close to $-\infty$. In particular, we prove that the two solitons survive the collision up to a correction of lower order, i.e. for all t large, we have

$$u(t,x) = Q_{\tilde{c}_1}(x - y_1(t)) + Q_{\tilde{c}_2}(x - y_2(t)) + \tilde{\eta}(t,x), \tag{3}$$

where $\tilde{c}_1 \sim c_1$, $\tilde{c}_2 \sim c_2$ and $\|\tilde{\eta}(t)\|_{H^1} \ll \|Q_{c_2}\|_{H^1}$. From an explicit decomposition in the interaction region, we can describe precisely the collision. For $p = 2$, 3, we recover classical results at the main orders. For $p = 4$, this description is completely new.

Second, our analysis in the nonintegrable case $p = 4$ proves that no pure 2-soliton solution exists in this regime ($c_2 \ll c_1$). This is clearly in contrast with the integrable case for which explicit multi-soliton solutions exist.

Nevertheless, we exhibit new exceptional solutions for $p = 4$ which are natural extensions in the nonintegrable case of the multi-solitons.