
* Princeton Discrete Math Seminar *

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Counting homomorphisms from Q_d to \mathbf{Z}

Abstract

Let Q_d be the usual discrete hypercube — the graph on vertex set $\{0, 1\}^d$ in which two strings are adjacent if they differ on exactly one coordinate. Write \mathcal{F} for the set of functions from $\{0, 1\}^d$ to \mathbf{Z} which map adjacent vertices to consecutive integers and map some fixed vertex to 0 (\mathcal{F} is the set of *homomorphisms* from Q_d to \mathbf{Z} , modded out by the obvious equivalence relation).

Recently we showed that if f is chosen uniformly from \mathcal{F} , then f almost surely takes on at most 5 values, confirming a conjecture of Kahn. This is equivalent to showing that the number of proper 3-colourings of Q_d is asymptotic to $6e2^{2^{d-1}}$. Our proof was in part inspired by earlier work of Korshunov and Sapozhenko, who showed that the number of independent sets in Q_d is asymptotic to $2\sqrt{e}2^{2^{d-1}}$.

In this talk, we will outline proofs of both these results.