Full Regularity of Variational Solutions to Two Phase Free Boundary Problems in 3-Dimensions

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Abstract:

We consider variational solutions to free boundary problems of the type, (for a function defined in the bounded smooth domain $\Omega \subset \mathbb{R}^n$, with boundary values a given smooth function g)

$$\begin{cases} \triangle u = 0 \quad \text{in } \{u \neq 0\} \cap \Omega \\ | \bigtriangledown u^+|^2 - | \bigtriangledown u^-|^2 = 1 \text{ on } \partial\{u > 0\} \cap \Omega, \end{cases}$$

where u^+, u^- are the positive and negative parts of u. The set $F(u) = \partial\{u > 0\} \subset \Omega$ is called the free boundary.

It has been known since the early 80's, from the work of Alt-Caffarelli-Friedman that when $n = 2$, $F(u)$ is a smooth curve. The corresponding result in higher dimensions was unkown. We have shown, with Caffarelli and Jerison that the analogous result still holds when $n = 3$. We have also shown that this still holds for variational solutions of

$$\begin{cases} \triangle u = f \quad \text{in } \{u \neq 0\} \cap \Omega \\ | \bigtriangledown u^+|^2 - | \bigtriangledown u^-|^2 = 1 \text{ on } \partial\{u > 0\} \cap \Omega, \end{cases}$$

with $f \epsilon L^\infty(\Omega)$. We will discuss these results and, time permitting, some recent work on how $F(u)$ meets $\partial\Omega$.