Proof of Theorem 98b

The theorem to be proved is

$x + 0 = 0 + x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x + 0) = (0 + x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ 0 + x = x + 0$ from H:x

1: $x + 0 = x$ from 12;x

2: $0 + x = x$ from 97;x

Equality substitutions:

3: $\neg\ x + 0 = x \quad \vee \quad 0 + x = x + 0 \quad \vee \quad \neg\ 0 + x = x$

Inferences:

4: $\neg\ x + 0 = x \quad \vee \quad \neg\ 0 + x = x$ by
0: $\neg\ 0 + x = x + 0$
3: $\neg\ x + 0 = x \quad \vee \quad 0 + x = x + 0 \quad \vee \quad \neg\ 0 + x = x$

5: $\neg\ 0 + x = x$ by
1: $x + 0 = x$
4: $\neg\ x + 0 = x \quad \vee \quad \neg\ 0 + x = x$

6: QEA by
2: $0 + x = x$
5: $\neg\ 0 + x = x$