## Proof of Theorem 97i

The theorem to be proved is

$0 + x = x \quad \rightarrow \quad 0 + \mathrm{S}x = \mathrm{S}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(0 + x) = (x)] \quad \& \quad [\neg\, (0 + (\mathrm{S}x)) = (\mathrm{S}x)]]$

### Special cases of the hypothesis and previous results:

0: $0 + x = x$ from H:$x$

1: $\neg\, 0 + (\mathrm{S}x) = \mathrm{S}x$ from H:$x$

2: $\mathrm{S}(0 + x) = 0 + (\mathrm{S}x)$ from 12;0;$x$

### Equality substitutions:

3: $\neg\, 0 + x = x \quad \vee \quad \neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x) \quad \vee \quad \mathrm{S}(x) = 0 + (\mathrm{S}x)$

### Inferences:

4: $\neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x) \quad \vee \quad 0 + (\mathrm{S}x) = \mathrm{S}x$ by
0: $0 + x = x$
3: $\neg\, 0 + x = x \quad \vee \quad \neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x) \quad \vee \quad 0 + (\mathrm{S}x) = \mathrm{S}x$

5: $\neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x)$ by
1: $\neg\, 0 + (\mathrm{S}x) = \mathrm{S}x$
4: $\neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x) \quad \vee \quad 0 + (\mathrm{S}x) = \mathrm{S}x$

6: $QEA$ by
2: $\mathrm{S}(0 + x) = 0 + (\mathrm{S}x)$
5: $\neg\, \mathrm{S}(0 + x) = 0 + (\mathrm{S}x)$