

Proof of Theorem 80

The theorem to be proved is

$$\neg [x < y \ \& \ y \leq x]$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[(x) < (y)] \ \& \ [(y) \leq (x)]]$$

Special cases of the hypothesis and previous results:

$$0: \ x < y \quad \text{from } H:x:y$$

$$1: \ y \leq x \quad \text{from } H:x:y$$

$$2: \ \neg x < y \ \vee \ \neg y \leq x \quad \text{from } \text{\color{blue}78};x;y$$

Inferences:

$$3: \ \neg y \leq x \quad \text{by}$$

$$0: \ x < y$$

$$2: \ \neg x < y \ \vee \ \neg y \leq x$$

$$4: \ QEA \quad \text{by}$$

$$1: \ y \leq x$$

$$3: \ \neg y \leq x$$