Proof of Theorem 79

The theorem to be proved is

$\neg\, y \leq x \quad \rightarrow \quad x < y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(y) \leq (x)] \quad \& \quad [\neg\,(x) < (y)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, y \leq x$ from H:y:x

1: $\neg\, x < y$ from H:y:x

2: $x < y \;\vee\; y = x \;\vee\; y < x$ from 77;x;y

3: $\neg\, y < x \;\vee\; y \leq x$ from $56^{\rightarrow}$;y;x

4: $x \leq x$ from 60;x

Equality substitutions:

5: $\neg\, y = x \;\vee\; y \leq x \;\vee\; \neg\, x \leq x$

Inferences:

6: $\neg\, y < x$ by
0: $\neg\, y \leq x$
3: $\neg\, y < x \;\vee\; y \leq x$

7: $\neg\, y = x \;\vee\; \neg\, x \leq x$ by
0: $\neg\, y \leq x$
5: $\neg\, y = x \;\vee\; y \leq x \;\vee\; \neg\, x \leq x$

8: $y = x \;\vee\; y < x$ by
1: $\neg\, x < y$
2: $x < y \;\vee\; y = x \;\vee\; y < x$

9: $\neg\, y = x$ by
4: $x \leq x$
7: $\neg\, y = x \;\vee\; \neg\, x \leq x$

10: $y = x$ by

6: $\neg\, y < x$

8: $y = x \quad \vee \quad y < x$

11: QEA by

9: $\neg\, y = x$

10: $y = x$