Proof of Theorem 78

The theorem to be proved is

$x < y \quad \rightarrow \quad \neg\, y \leq x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad [[(x) < (y)] \quad \& \quad [(y) \leq (x)]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x:y

1: $y \leq x$ from H:x:y

2: $\neg\, x < y \quad \vee \quad x \leq y$ from 56$^{\rightarrow}$;x;y

3: $\neg\, x < y \quad \vee \quad \neg\, y = x$ from 56$^{\rightarrow}$;x;y

4: $\neg\, x \leq y \quad \vee \quad \neg\, y \leq x \quad \vee \quad y = x$ from 76;x;y

Inferences:

5: $x \leq y$ by
0: $x < y$
2: $\neg\, x < y \quad \vee \quad x \leq y$

6: $\neg\, y = x$ by
0: $x < y$
3: $\neg\, x < y \quad \vee \quad \neg\, y = x$

7: $\neg\, x \leq y \quad \vee \quad y = x$ by
1: $y \leq x$
4: $\neg\, x \leq y \quad \vee \quad \neg\, y \leq x \quad \vee \quad y = x$

8: $y = x$ by
5: $x \leq y$
7: $\neg\, x \leq y \quad \vee \quad y = x$

9: QEA by
6: $\neg\, y = x$
8: $y = x$