Proof of Theorem 77

The theorem to be proved is

$x < y \quad \vee \quad x = y \quad \vee \quad y < x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) < (y)] \quad \& \quad [\neg\,(x) = (y)] \quad \& \quad [\neg\,(y) < (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x < y$ from H:x:y

1: $\neg\, y = x$ from H:x:y

2: $\neg\, y < x$ from H:x:y

3: $x - y = 0 \quad \vee \quad y < x$ from 75;x;y

4: $y - x = 0 \quad \vee \quad x < y$ from 75;y;x

5: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad y = x$ from 30;x;y

Inferences:

6: $y - x = 0$ by

0: $\neg\, x < y$

4: $y - x = 0 \quad \vee \quad x < y$

7: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0$ by

1: $\neg\, y = x$

5: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad y = x$

8: $x - y = 0$ by

2: $\neg\, y < x$

3: $x - y = 0 \quad \vee \quad y < x$

9: $\neg\, x - y = 0$ by

6: $y - x = 0$

7: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0$

10: QEA by

8: $x - y = 0$

9: $\neg\, x - y = 0$