Proof of Theorem 76

The theorem to be proved is

$x \leq y \quad \& \quad y \leq x \quad \rightarrow \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \leq (y)] \quad \& \quad [(y) \leq (x)] \quad \& \quad [\neg\, (x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $x \leq y$ from H:x:y

1: $y \leq x$ from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $\neg\, x \leq y \quad \vee \quad x - y = 0$ from 55$^{\rightarrow}$;x;y

4: $\neg\, y \leq x \quad \vee \quad y - x = 0$ from 55$^{\rightarrow}$;y;x

5: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad y = x$ from 30;x;y

Inferences:

6: $x - y = 0$ by
0: $x \leq y$
3: $\neg\, x \leq y \quad \vee \quad x - y = 0$

7: $y - x = 0$ by
1: $y \leq x$
4: $\neg\, y \leq x \quad \vee \quad y - x = 0$

8: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0$ by
2: $\neg\, y = x$
5: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad y = x$

9: $\neg\, y - x = 0$ by
6: $x - y = 0$
8: $\neg\, x - y = 0 \quad \vee \quad \neg\, y - x = 0$

10: QEA by
7: $y - x = 0$
9: $\neg\, y - x = 0$