Proof of Theorem 75

The theorem to be proved is

$y - x \neq 0 \quad \rightarrow \quad x < y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (y - x) = (0)] \quad \& \quad [\neg\ (x) < (y)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ y - x = 0$ from H:y:x

1: $\neg\ x < y$ from H:y:x

2: $y - x = 0 \quad \vee \quad x + (y - x) = y$ from 23;y;x

3: $x \leq x + (y - x)$ from 71;x;$y - x$

4: $x < y \quad \vee \quad \neg\ x \leq y \quad \vee \quad y = x$ from 56$^{\Leftarrow}$;x;y

5: $x - x = 0$ from 19;x

Equality substitutions:

6: $\neg\ x + (y - x) = y \quad \vee \quad \neg\ x \leq x + (y - x) \quad \vee \quad x \leq y$

7: $\neg\ y = x \quad \vee \quad y - x = 0 \quad \vee \quad \neg\ x - x = 0$

Inferences:

8: $x + (y - x) = y$ by
0: $\neg\ y - x = 0$
2: $y - x = 0 \quad \vee \quad x + (y - x) = y$

9: $\neg\ y = x \quad \vee \quad \neg\ x - x = 0$ by
0: $\neg\ y - x = 0$
7: $\neg\ y = x \quad \vee \quad y - x = 0 \quad \vee \quad \neg\ x - x = 0$

10: $\neg\ x \leq y \quad \vee \quad y = x$ by
1: $\neg\ x < y$
4: $x < y \quad \vee \quad \neg\ x \leq y \quad \vee \quad y = x$

11: $\neg\, x + (y - x) = y \quad \vee \quad x \leq y$ by

3: $x \leq x + (y - x)$

6: $\neg\, x + (y - x) = y \quad \vee \quad \neg\, x \leq x + (y - x) \quad \vee \quad x \leq y$

12: $\neg\, y = x$ by

5: $x - x = 0$

9: $\neg\, y = x \quad \vee \quad \neg\, x - x = 0$

13: $x \leq y$ by

8: $x + (y - x) = y$

11: $\neg\, x + (y - x) = y \quad \vee \quad x \leq y$

14: $\neg\, x \leq y$ by

12: $\neg\, y = x$

10: $\neg\, x \leq y \quad \vee \quad y = x$

15: QEA by

13: $x \leq y$

14: $\neg\, x \leq y$