

Proof of Theorem 74

The theorem to be proved is

$$x < y \rightarrow y - x \neq 0$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [(x) < (y)] \quad \& \quad [(y - x) = (0)]$$

Special cases of the hypothesis and previous results:

- 0: $x < y$ from H: $x:y$
- 1: $y - x = 0$ from H: $x:y$
- 2: $\neg x < y \vee x \leq y$ from [56](#)[>]; $x;y$
- 3: $\neg x < y \vee \neg y = x$ from [56](#)[>]; $x;y$
- 4: $\neg x \leq y \vee x + (y - x) = y$ from [68](#); $x;y$
- 5: $x + 0 = x$ from [12](#); x

Equality substitutions:

- 6: $\neg y - x = 0 \vee \neg x + (y - x) = y \vee x + (0) = y$
- 7: $\neg x + 0 = x \vee \neg x + 0 = y \vee x = y$

Inferences:

- 8: $x \leq y$ by
 - 0: $x < y$
 - 2: $\neg x < y \vee x \leq y$
- 9: $\neg y = x$ by
 - 0: $x < y$
 - 3: $\neg x < y \vee \neg y = x$
- 10: $\neg x + (y - x) = y \vee x + 0 = y$ by
 - 1: $y - x = 0$
 - 6: $\neg y - x = 0 \vee \neg x + (y - x) = y \vee x + 0 = y$

- 11: $\neg x + 0 = y \vee y = x$ by
 5: $x + 0 = x$
 7: $\neg x + 0 = x \vee \neg x + 0 = y \vee y = x$
- 12: $x + (y - x) = y$ by
 8: $x \leq y$
 4: $\neg x \leq y \vee x + (y - x) = y$
- 13: $\neg x + 0 = y$ by
 9: $\neg y = x$
 11: $\neg x + 0 = y \vee y = x$
- 14: $x + 0 = y$ by
 12: $x + (y - x) = y$
 10: $\neg x + (y - x) = y \vee x + 0 = y$
- 15: *QEA* by
 13: $\neg x + 0 = y$
 14: $x + 0 = y$