Proof of Theorem 72b

The theorem to be proved is

$x + (y + 0) = (x + y) + 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x + (y + 0)) = ((x + y) + 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ x + (y + 0) = (x + y) + 0$ from H:x:y

1: $y + 0 = y$ from 12;y

2: $(x + y) + 0 = x + y$ from 12;$x + y$

Equality substitutions:

3: $\neg\ y + 0 = y \quad \vee \quad x + (y + 0) = (x + y) + 0 \quad \vee \quad \neg\ x + (y) = (x + y) + 0$

Inferences:

4: $\neg\ y + 0 = y \quad \vee \quad \neg\ (x + y) + 0 = x + y$ by
0: $\neg\ x + (y + 0) = (x + y) + 0$
3: $\neg\ y + 0 = y \quad \vee \quad x + (y + 0) = (x + y) + 0 \quad \vee \quad \neg\ (x + y) + 0 = x + y$

5: $\neg\ (x + y) + 0 = x + y$ by
1: $y + 0 = y$
4: $\neg\ y + 0 = y \quad \vee \quad \neg\ (x + y) + 0 = x + y$

6: QEA by
2: $(x + y) + 0 = x + y$
5: $\neg\ (x + y) + 0 = x + y$