Proof of Theorem 71

The theorem to be proved is

$x \leq x + y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) \leq (x + y)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \leq x + y$ from H:x:y

1: $x - (x + y) = (x - x) - y$ from 69;x;x;y

2: $x - x = 0$ from 19;x

3: $0 - y = 0$ from 70;y

4: $x \leq x + y \quad \vee \quad \neg\, x - (x + y) = 0$ from $55^{\leftarrow}$;x;$x + y$

Equality substitutions:

5: $\neg\, x - x = 0 \quad \vee \quad \neg\, x - (x + y) = (x - x) - y \quad \vee \quad x - (x + y) = (0) - y$

6: $\neg\, 0 - y = 0 \quad \vee \quad \neg\, x - (x + y) = 0 - y \quad \vee \quad x - (x + y) = 0$

Inferences:

7: $\neg\, x - (x + y) = 0$ by
0: $\neg\, x \leq x + y$
4: $x \leq x + y \quad \vee \quad \neg\, x - (x + y) = 0$

8: $\neg\, x - x = 0 \quad \vee \quad x - (x + y) = 0 - y$ by
1: $x - (x + y) = (x - x) - y$
5: $\neg\, x - x = 0 \quad \vee \quad \neg\, x - (x + y) = (x - x) - y \quad \vee \quad x - (x + y) = 0 - y$

9: $x - (x + y) = 0 - y$ by
2: $x - x = 0$
8: $\neg\, x - x = 0 \quad \vee \quad x - (x + y) = 0 - y$

10: $\neg\, x - (x + y) = 0 - y \quad \vee \quad x - (x + y) = 0$ by
3: $0 - y = 0$
6: $\neg\, 0 - y = 0 \quad \vee \quad \neg\, x - (x + y) = 0 - y \quad \vee \quad x - (x + y) = 0$

11: $\neg\ x-(x+y)=0-y$ by

7: $\neg\ x-(x+y)=0$

10: $\neg\ x-(x+y)=0-y \quad \vee \quad x-(x+y)=0$

12: QEA by

9: $x-(x+y)=0-y$

11: $\neg\ x-(x+y)=0-y$