Proof of Theorem 70i

The theorem to be proved is

$0 - x = 0 \quad \rightarrow \quad 0 - \mathrm{S}x = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $\quad [[(0 - x) = (0)] \quad \& \quad [\neg\,(0 - (\mathrm{S}x)) = (0)]]$

Special cases of the hypothesis and previous results:

0: $0 - x = 0$ from H:x

1: $\neg\, 0 - (\mathrm{S}x) = 0$ from H:x

2: $\mathrm{P}(0 - x) = 0 - (\mathrm{S}x)$ from 17;0;x

3: $\mathrm{P}0 = 0$ from 16

Equality substitutions:

4: $\neg\, 0 - x = 0 \quad \vee \quad \neg\, \mathrm{P}(0 - x) = 0 - (\mathrm{S}x) \quad \vee \quad \mathrm{P}(0) = 0 - (\mathrm{S}x)$

5: $\neg\, \mathrm{P}0 = 0 \quad \vee \quad \neg\, 0 - (\mathrm{S}x) = \mathrm{P}0 \quad \vee \quad 0 - (\mathrm{S}x) = 0$

Inferences:

6: $\neg\, \mathrm{P}(0 - x) = 0 - (\mathrm{S}x) \quad \vee \quad 0 - (\mathrm{S}x) = \mathrm{P}0$ by
0: $0 - x = 0$
4: $\neg\, 0 - x = 0 \quad \vee \quad \neg\, \mathrm{P}(0 - x) = 0 - (\mathrm{S}x) \quad \vee \quad 0 - (\mathrm{S}x) = \mathrm{P}0$

7: $\neg\, \mathrm{P}0 = 0 \quad \vee \quad \neg\, 0 - (\mathrm{S}x) = \mathrm{P}0$ by
1: $\neg\, 0 - (\mathrm{S}x) = 0$
5: $\neg\, \mathrm{P}0 = 0 \quad \vee \quad \neg\, 0 - (\mathrm{S}x) = \mathrm{P}0 \quad \vee \quad 0 - (\mathrm{S}x) = 0$

8: $0 - (\mathrm{S}x) = \mathrm{P}0$ by
2: $\mathrm{P}(0 - x) = 0 - (\mathrm{S}x)$
6: $\neg\, \mathrm{P}(0 - x) = 0 - (\mathrm{S}x) \quad \vee \quad 0 - (\mathrm{S}x) = \mathrm{P}0$

9: $\neg\, 0 - (\mathrm{S}x) = \mathrm{P}0$ by
3: $\mathrm{P}0 = 0$
7: $\neg\, \mathrm{P}0 = 0 \quad \vee \quad \neg\, 0 - (\mathrm{S}x) = \mathrm{P}0$

10: QEA by
8: $0 - (\mathrm{S}x) = \mathrm{P}0$
9: $\neg\, 0 - (\mathrm{S}x) = \mathrm{P}0$