Proof of Theorem 69b

The theorem to be proved is

$x - (y + 0) = (x - y) - 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x - (y + 0)) = ((x - y) - 0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x - (y + 0) = (x - y) - 0$ from H:x:y

1: $y + 0 = y$ from 12;y

2: $(x - y) - 0 = x - y$ from 17;$x - y$

Equality substitutions:

3: $\neg\, y + 0 = y \quad \vee \quad x - (y + 0) = (x - y) - 0 \quad \vee \quad \neg\, x - (y) = (x - y) - 0$

Inferences:

4: $\neg\, y + 0 = y \quad \vee \quad \neg\, (x - y) - 0 = x - y$ by
0: $\neg\, x - (y + 0) = (x - y) - 0$
3: $\neg\, y + 0 = y \quad \vee \quad x - (y + 0) = (x - y) - 0 \quad \vee \quad \neg\, (x - y) - 0 = x - y$

5: $\neg\, (x - y) - 0 = x - y$ by
1: $y + 0 = y$
4: $\neg\, y + 0 = y \quad \vee \quad \neg\, (x - y) - 0 = x - y$

6: QEA by
2: $(x - y) - 0 = x - y$
5: $\neg\, (x - y) - 0 = x - y$