Proof of Theorem 68

The theorem to be proved is

$x \leq y \quad \rightarrow \quad y = x + (y - x)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \leq (y)] \quad \& \quad [\neg\ (y) = (x + (y - x))]]$

Special cases of the hypothesis and previous results:

0: $x \leq y$ from H:x:y

1: $\neg\ x + (y - x) = y$ from H:x:y

2: $x < y \quad \vee \quad \neg\ x \leq y \quad \vee \quad y = x$ from 56$^{\Leftarrow}$;x;y

3: $\neg\ y = x \quad \vee \quad x + (y - x) = y$ from 67;x;y

4: $\neg\ x < y \quad \vee \quad x + (y - x) = y$ from 66;x;y

Inferences:

5: $x < y \quad \vee \quad y = x$ by
0: $x \leq y$
2: $x < y \quad \vee \quad \neg\ x \leq y \quad \vee \quad y = x$

6: $\neg\ y = x$ by
1: $\neg\ x + (y - x) = y$
3: $\neg\ y = x \quad \vee \quad x + (y - x) = y$

7: $\neg\ x < y$ by
1: $\neg\ x + (y - x) = y$
4: $\neg\ x < y \quad \vee \quad x + (y - x) = y$

8: $x < y$ by
6: $\neg\ y = x$
5: $x < y \quad \vee \quad y = x$

9: QEA by
7: $\neg\ x < y$
8: $x < y$