Proof of Theorem 67

The theorem to be proved is

$x = y \quad \rightarrow \quad y = x + (y - x)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) = (y)] \quad \& \quad [\neg\ (y) = (x + (y - x))]]$

Special cases of the hypothesis and previous results:

0: $y = x$ from H:x:y

1: $\neg\ x + (y - x) = y$ from H:x:y

2: $y - y = 0$ from 19;y

3: $x + 0 = x$ from 12;x

Equality substitutions:

4: $\neg\ y = x \quad \vee \quad x + (y - x) = y \quad \vee \quad \neg\ x + (x - x) = x$

5: $\neg\ y = x \quad \vee \quad \neg\ y - y = 0 \quad \vee \quad x - x = 0$

6: $\neg\ x - x = 0 \quad \vee \quad x + (x - x) = x \quad \vee \quad \neg\ x + (0) = x$

Inferences:

7: $x + (y - x) = y \quad \vee \quad \neg\ x + (x - x) = x$ by
0: $y = x$
4: $\neg\ y = x \quad \vee \quad x + (y - x) = y \quad \vee \quad \neg\ x + (x - x) = x$

8: $\neg\ y - y = 0 \quad \vee \quad x - x = 0$ by
0: $y = x$
5: $\neg\ y = x \quad \vee \quad \neg\ y - y = 0 \quad \vee \quad x - x = 0$

9: $\neg\ x + (x - x) = x$ by
1: $\neg\ x + (y - x) = y$
7: $x + (y - x) = y \quad \vee \quad \neg\ x + (x - x) = x$

10: $x - x = 0$ by
2: $y - y = 0$
8: $\neg\ y - y = 0 \quad \vee \quad x - x = 0$

11: $\neg\, x - x = 0 \quad \vee \quad x + (x - x) = x$ by

3: $x + 0 = x$

6: $\neg\, x - x = 0 \quad \vee \quad x + (x - x) = x \quad \vee \quad \neg\, x + 0 = x$

12: $\neg\, x - x = 0$ by

9: $\neg\, x + (x - x) = x$

11: $\neg\, x - x = 0 \quad \vee \quad x + (x - x) = x$

13: QEA by

10: $x - x = 0$

12: $\neg\, x - x = 0$