Proof of Theorem 66

The theorem to be proved is

$x < y \quad \rightarrow \quad y = x + (y - x)$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) < (y)] \quad \& \quad [\neg\, (y) = (x + (y - x))]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x:y

1: $\neg\, x + (y - x) = y$ from H:x:y

2: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$ from 65;x;y

3: $y - x = 0 \quad \vee \quad x + (y - x) = y$ from 23;y;x

Inferences:

4: $\neg\, y - x = 0$ by
 0: $x < y$
 2: $\neg\, x < y \quad \vee \quad \neg\, y - x = 0$

5: $y - x = 0$ by
 1: $\neg\, x + (y - x) = y$
 3: $y - x = 0 \quad \vee \quad x + (y - x) = y$

6: QEA by
 4: $\neg\, y - x = 0$
 5: $y - x = 0$