Proof of Theorem 65

The theorem to be proved is

$x < y \quad \rightarrow \quad y - x \neq 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) < (y)] \quad \& \quad [(y - x) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x < y$ from H:x:y

1: $y - x = 0$ from H:x:y

2: $\neg\, x < y \quad \vee \quad x \leq y$ from 56$^{\rightarrow}$;x;y

3: $\neg\, x < y \quad \vee \quad \neg\, y = x$ from 56$^{\rightarrow}$;x;y

4: $\neg\, x \leq y \quad \vee \quad x - y = 0$ from 55$^{\rightarrow}$;x;y

5: $y = x \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad \neg\, x - y = 0$ from 29;x;y

Inferences:

6: $x \leq y$ by
- 0: $x < y$
- 2: $\neg\, x < y \quad \vee \quad x \leq y$

7: $\neg\, y = x$ by
- 0: $x < y$
- 3: $\neg\, x < y \quad \vee \quad \neg\, y = x$

8: $y = x \quad \vee \quad \neg\, x - y = 0$ by
- 1: $y - x = 0$
- 5: $y = x \quad \vee \quad \neg\, y - x = 0 \quad \vee \quad \neg\, x - y = 0$

9: $x - y = 0$ by
- 6: $x \leq y$
- 4: $\neg\, x \leq y \quad \vee \quad x - y = 0$

10: $\neg\, x - y = 0$ by
- 7: $\neg\, y = x$
- 8: $y = x \quad \vee \quad \neg\, x - y = 0$

11: QEA by
- 9: $x - y = 0$
- 10: $\neg\, x - y = 0$