Proof of Theorem 64i

The theorem to be proved is

$\mathrm{P}x \leq x \quad \rightarrow \quad \mathrm{PS}x \leq \mathrm{S}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\mathrm{P}x) \leq (x)] \quad \& \quad [\neg\,(\mathrm{P}(\mathrm{S}x)) \leq (\mathrm{S}x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\,\mathrm{P}(\mathrm{S}x) \leq \mathrm{S}x$ from H:x

1: $\mathrm{P}(\mathrm{S}x) = x$ from 16;x

2: $x \leq \mathrm{S}x$ from 63;x

Equality substitutions:

3: $\neg\,\mathrm{P}(\mathrm{S}x) = x \quad \vee \quad \mathrm{P}(\mathrm{S}x) \leq \mathrm{S}x \quad \vee \quad \neg\, x \leq \mathrm{S}x$

Inferences:

4: $\neg\,\mathrm{P}(\mathrm{S}x) = x \quad \vee \quad \neg\, x \leq \mathrm{S}x$ by
0: $\neg\,\mathrm{P}(\mathrm{S}x) \leq \mathrm{S}x$
3: $\neg\,\mathrm{P}(\mathrm{S}x) = x \quad \vee \quad \mathrm{P}(\mathrm{S}x) \leq \mathrm{S}x \quad \vee \quad \neg\, x \leq \mathrm{S}x$

5: $\neg\, x \leq \mathrm{S}x$ by
1: $\mathrm{P}(\mathrm{S}x) = x$
4: $\neg\,\mathrm{P}(\mathrm{S}x) = x \quad \vee \quad \neg\, x \leq \mathrm{S}x$

6: QEA by
2: $x \leq \mathrm{S}x$
5: $\neg\, x \leq \mathrm{S}x$