## Proof of Theorem 63

The theorem to be proved is

$x \leq \mathrm{S}x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\,(x) \leq (\mathrm{S}x)]]$

### Special cases of the hypothesis and previous results:

0: $\neg\, x \leq \mathrm{S}x$ from H:$x$

1: $\mathrm{P}(x - x) = x - (\mathrm{S}x)$ from 17;$x$;$x$

2: $x - x = 0$ from 19;$x$

3: $\mathrm{P}0 = 0$ from 16

4: $x \leq \mathrm{S}x \;\vee\; \neg\, x - (\mathrm{S}x) = 0$ from $55^{\Leftarrow}$;$x$;S$x$

### Equality substitutions:

5: $\neg\, x - x = 0 \;\vee\; \neg\, \mathrm{P}(x - x) = x - (\mathrm{S}x) \;\vee\; \mathrm{P}(0) = x - (\mathrm{S}x)$

6: $\neg\, \mathrm{P}0 = 0 \;\vee\; \neg\, x - (\mathrm{S}x) = \mathrm{P}0 \;\vee\; x - (\mathrm{S}x) = 0$

### Inferences:

7: $\neg\, x - (\mathrm{S}x) = 0$ by
0: $\neg\, x \leq \mathrm{S}x$
4: $x \leq \mathrm{S}x \;\vee\; \neg\, x - (\mathrm{S}x) = 0$

8: $\neg\, x - x = 0 \;\vee\; x - (\mathrm{S}x) = \mathrm{P}0$ by
1: $\mathrm{P}(x - x) = x - (\mathrm{S}x)$
5: $\neg\, x - x = 0 \;\vee\; \neg\, \mathrm{P}(x - x) = x - (\mathrm{S}x) \;\vee\; x - (\mathrm{S}x) = \mathrm{P}0$

9: $x - (\mathrm{S}x) = \mathrm{P}0$ by
2: $x - x = 0$
8: $\neg\, x - x = 0 \;\vee\; x - (\mathrm{S}x) = \mathrm{P}0$

10: $\neg\, x - (\mathrm{S}x) = \mathrm{P}0 \;\vee\; x - (\mathrm{S}x) = 0$ by
3: $\mathrm{P}0 = 0$
6: $\neg\, \mathrm{P}0 = 0 \;\vee\; \neg\, x - (\mathrm{S}x) = \mathrm{P}0 \;\vee\; x - (\mathrm{S}x) = 0$

11: $\neg\, x - (\mathrm{S}x) = \mathrm{P}0$ by

7: $\neg\, x - (\mathrm{S}x) = 0$

10: $\neg\, x - (\mathrm{S}x) = \mathrm{P}0 \quad \vee \quad x - (\mathrm{S}x) = 0$

12: $QEA$ by

9: $x - (\mathrm{S}x) = \mathrm{P}0$

11: $\neg\, x - (\mathrm{S}x) = \mathrm{P}0$