Proof of Theorem 62

The theorem to be proved is

$x < y \quad \vee \quad x = y \quad \rightarrow \quad x \leq y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) < (y) \quad \vee \quad (x) = (y)] \quad \& \quad [\neg\, (x) \leq (y)]]$

Special cases of the hypothesis and previous results:

0: $x < y \quad \vee \quad y = x$ from H:x:y

1: $\neg\, x \leq y$ from H:x:y

2: $\neg\, x < y \quad \vee \quad x \leq y$ from 56$^{\rightarrow}$;x;y

3: $x \leq x$ from 60;x

Equality substitutions:

4: $\neg\, y = x \quad \vee \quad x \leq y \quad \vee \quad \neg\, x \leq x$

Inferences:

5: $\neg\, x < y$ by
1: $\neg\, x \leq y$
2: $\neg\, x < y \quad \vee \quad x \leq y$

6: $\neg\, y = x \quad \vee \quad \neg\, x \leq x$ by
1: $\neg\, x \leq y$
4: $\neg\, y = x \quad \vee \quad x \leq y \quad \vee \quad \neg\, x \leq x$

7: $\neg\, y = x$ by
3: $x \leq x$
6: $\neg\, y = x \quad \vee \quad \neg\, x \leq x$

8: $y = x$ by
5: $\neg\, x < y$
0: $x < y \quad \vee \quad y = x$

9: QEA by
7: $\neg\, y = x$
8: $y = x$