Proof of Theorem 61

The theorem to be proved is

$x \leq y \quad \rightarrow \quad x < y \quad \vee \quad x = y$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \leq (y)] \quad \& \quad [\neg\,(x) < (y)] \quad \& \quad [\neg\,(x) = (y)]]$

Special cases of the hypothesis and previous results:

0: $x \leq y$ from H:x:y

1: $\neg\, x < y$ from H:x:y

2: $\neg\, y = x$ from H:x:y

3: $x < y \quad \vee \quad \neg\, x \leq y \quad \vee \quad y = x$ from $56^{\Leftarrow}$;x;y

Inferences:

4: $x < y \quad \vee \quad y = x$ by
0: $x \leq y$
3: $x < y \quad \vee \quad \neg\, x \leq y \quad \vee \quad y = x$

5: $y = x$ by
1: $\neg\, x < y$
4: $x < y \quad \vee \quad y = x$

6: QEA by
2: $\neg\, y = x$
5: $y = x$