## Proof of Theorem 59

The theorem to be proved is

$$\neg Sx \leq x$$

Suppose the theorem does not hold. Then, with the variables held fixed,

$$(H) \quad [[(Sx) \le (x)]]$$

## Special cases of the hypothesis and previous results:

- 0:  $Sx \le x$  from H:x
- 1:  $\neg Sx \le x \lor (Sx) x = 0$  from  $\underline{55} \Rightarrow ;Sx;x$
- 2:  $\neg (Sx) x = 0$  from  $\underline{21};x$

## **Inferences:**

- 3: (Sx) x = 0 by
  - $0: Sx \leq x$
  - 1:  $\neg Sx \le x \lor (Sx) x = 0$
- 4: QEA by
  - 2:  $\neg (Sx) x = 0$
  - 3: (Sx) x = 0