Proof of Theorem 58

The theorem to be proved is

$0 \leq x$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (0) \leq (x)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ 0 \leq x$ from H:x

1: $0 \leq x \quad \vee \quad \neg\ 0 - x = 0$ from 55$^{\Leftarrow}$;0;x

2: $0 - x = 0 \quad \vee \quad x + (0 - x) = 0$ from 23;0;x

3: $\neg\ x + (0 - x) = 0 \quad \vee \quad 0 - x = 0$ from 15;x;$0 - x$

Inferences:

4: $\neg\ 0 - x = 0$ by

0: $\neg\ 0 \leq x$

1: $0 \leq x \quad \vee \quad \neg\ 0 - x = 0$

5: $x + (0 - x) = 0$ by

4: $\neg\ 0 - x = 0$

2: $0 - x = 0 \quad \vee \quad x + (0 - x) = 0$

6: $\neg\ x + (0 - x) = 0$ by

4: $\neg\ 0 - x = 0$

3: $\neg\ x + (0 - x) = 0 \quad \vee \quad 0 - x = 0$

7: QEA by

5: $x + (0 - x) = 0$

6: $\neg\ x + (0 - x) = 0$