Proof of Theorem 57

The theorem to be proved is

$x \leq 0 \quad \rightarrow \quad x = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) \leq (0)] \quad \& \quad [\neg\, (x) = (0)]]$

Special cases of the hypothesis and previous results:

0: $x \leq 0$ from H:x

1: $\neg\, 0 = x$ from H:x

2: $\neg\, x \leq 0 \quad \vee \quad x - 0 = 0$ from 55$^{\rightarrow}$;x;0

3: $x - 0 = x$ from 17;x

Equality substitutions:

4: $\neg\, x - 0 = 0 \quad \vee \quad \neg\, x - 0 = x \quad \vee \quad 0 = x$

Inferences:

5: $x - 0 = 0$ by
0: $x \leq 0$
2: $\neg\, x \leq 0 \quad \vee \quad x - 0 = 0$

6: $\neg\, x - 0 = 0 \quad \vee \quad \neg\, x - 0 = x$ by
1: $\neg\, 0 = x$
4: $\neg\, x - 0 = 0 \quad \vee \quad \neg\, x - 0 = x \quad \vee \quad 0 = x$

7: $\neg\, x - 0 = 0$ by
3: $x - 0 = x$
6: $\neg\, x - 0 = 0 \quad \vee \quad \neg\, x - 0 = x$

8: QEA by
5: $x - 0 = 0$
7: $\neg\, x - 0 = 0$