Proof of Theorem 47

The theorem to be proved is

$x \dot\vee y = 0 \quad \vee \quad x \dot\vee y = \mathrm{S}0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\, (x \dot\vee y) = (0)] \quad \& \quad [\neg\, (x \dot\vee y) = (\mathrm{S}0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\, x \dot\vee y = 0$ from H:x:y

1: $\neg\, x \dot\vee y = \mathrm{S}0$ from H:x:y

2: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y$ from 44;x;y

3: $\mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((y, 0, \mathrm{S}0)) = \mathrm{S}0$ from 38;y

4: $\mathrm{C}((x, 0, 0)) = 0$ from 40;x

5: $\mathrm{C}((x, 0, \mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$ from 38;x

Equality substitutions:

6: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad x \dot\vee y = 0$

7: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{S}0 \quad \vee \quad x \dot\vee y = \mathrm{S}0$

8: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, 0)) = 0$

9: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = 0$

10: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{S}0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{S}0)) = \mathrm{S}0$

Inferences:

11: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$ by
0: $\neg\, x \dot\vee y = 0$
6: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad x \dot\vee y = 0$

12: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{S}0$ by
1: $\neg\, x \dot\vee y = \mathrm{S}0$
7: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = x \dot\vee y \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{S}0 \quad \vee \quad x \dot\vee y = \mathrm{S}0$

13: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$ by

2: $\mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = x \mathbin{\dot\vee} y$

11: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = x \mathbin{\dot\vee} y \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$

14: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = \mathrm{S}0$ by

2: $\mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = x \mathbin{\dot\vee} y$

12: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = x \mathbin{\dot\vee} y \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = \mathrm{S}0$

15: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$ by

4: $\mathrm{C}((x,0,0)) = 0$

8: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0 \quad \vee \quad \neg \, \mathrm{C}((x,0,0)) = 0$

16: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = 0$ by

13: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$

9: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = 0$

17: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = 0$ by

13: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$

15: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = 0$

18: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$ by

14: $\neg \, \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = \mathrm{S}0$

10: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{C}((y,0,\mathrm{S}0)))) = \mathrm{S}0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$

19: $\mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0$ by

17: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = 0$

3: $\mathrm{C}((y,0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0$

20: $\neg \, \mathrm{C}((x,0,\mathrm{S}0)) = 0$ by

19: $\mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0$

16: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = 0$

21: $\neg \, \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$ by

19: $\mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0$

18: $\neg \, \mathrm{C}((y,0,\mathrm{S}0)) = \mathrm{S}0 \quad \vee \quad \neg \, \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$

22: $\mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$ by

20: $\neg \, \mathrm{C}((x,0,\mathrm{S}0)) = 0$

5: $\mathrm{C}((x,0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$

23: QEA by

21: $\neg \, \mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$

22: $\mathrm{C}((x,0,\mathrm{S}0)) = \mathrm{S}0$