Proof of Theorem 46

The theorem to be proved is

$\dot{\neg}x = 0 \quad \vee \quad \dot{\neg}x = \mathrm{S}0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (\dot{\neg}x) = (0)] \quad \& \quad [\neg\ (\dot{\neg}x) = (\mathrm{S}0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ \dot{\neg}x = 0$ from H:x

1: $\neg\ \dot{\neg}x = \mathrm{S}0$ from H:x

2: $\mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x$ from 43;x

3: $\mathrm{C}((x, \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$ from 39;x

Equality substitutions:

4: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0 \quad \vee \quad \dot{\neg}x = 0$

5: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \dot{\neg}x = \mathrm{S}0$

Inferences:

6: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0$ by
0: $\neg\ \dot{\neg}x = 0$
4: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0 \quad \vee \quad \dot{\neg}x = 0$

7: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$ by
1: $\neg\ \dot{\neg}x = \mathrm{S}0$
5: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0 \quad \vee \quad \dot{\neg}x = \mathrm{S}0$

8: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0$ by
2: $\mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x$
6: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0$

9: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$ by
2: $\mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x$
7: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \dot{\neg}x \quad \vee \quad \neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$

10: $\mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$ by

8: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = 0$

3: $\mathrm{C}((x, \mathrm{S}0, 0)) = 0 \quad \vee \quad \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$

11: QEA by

9: $\neg\ \mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$

10: $\mathrm{C}((x, \mathrm{S}0, 0)) = \mathrm{S}0$