Proof of Theorem 45

The theorem to be proved is

$x \dot{-} y = 0 \quad \lor \quad x \dot{-} y = \mathrm{S}0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (x \dot{-} y) = (0)] \quad \& \quad [\neg\ (x \dot{-} y) = (\mathrm{S}0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ x \dot{-} y = 0$ from H:x:y

1: $\neg\ x \dot{-} y = \mathrm{S}0$ from H:x:y

2: $\mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y$ from 42;x;y

3: $\mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0 \quad \lor \quad \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0$ from 38;Eq((x, y))

Equality substitutions:

4: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0 \quad \lor \quad x \dot{-} y = 0$

5: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0 \quad \lor \quad x \dot{-} y = \mathrm{S}0$

Inferences:

6: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0$ by
0: $\neg\ x \dot{-} y = 0$
4: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0 \quad \lor \quad x \dot{-} y = 0$

7: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0$ by
1: $\neg\ x \dot{-} y = \mathrm{S}0$
5: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0 \quad \lor \quad x \dot{-} y = \mathrm{S}0$

8: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0$ by
2: $\mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y$
6: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = 0$

9: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0$ by
2: $\mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y$
7: $\neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = x \dot{-} y \quad \lor \quad \neg\ \mathrm{C}((\mathrm{Eq}((x, y)), 0, \mathrm{S}0)) = \mathrm{S}0$

10: $\mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = \mathrm{S}0$ by

8: $\neg\ \mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = 0$

3: $\mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = 0 \quad \vee \quad \mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = \mathrm{S}0$

11: QEA by

9: $\neg\ \mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = \mathrm{S}0$

10: $\mathrm{C}((\mathrm{Eq}((x,y)),0,\mathrm{S}0)) = \mathrm{S}0$