Proof of Theorem 41b

The theorem to be proved is

$x = 0 \quad \vee \quad y = 0 \quad \rightarrow \quad \mathrm{C}\big(x, 0, \mathrm{C}(y, 0, \mathrm{S}0)\big) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(x) = (0) \quad \vee \quad (y) = (0)] \quad \& \quad [\neg\, (\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0))))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $0 = x \quad \vee \quad 0 = y$ from H:x:y

1: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$ from H:x:y

2: $\mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad \neg\, 0 = y$ from 36;y

3: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$ from 22;x

4: $\mathrm{C}((0, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$ from 33;0;C((y, 0, S0));Px

5: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ from 33;0;C((y, 0, S0));Px

Equality substitutions:

6: $\neg\, \mathrm{S}(\mathrm{P}x) = x \ \vee \ \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \ \vee \ \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

7: $\neg\ \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

8: $\neg\ x = 0 \quad \vee \quad \mathrm{C}(((x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\ \mathrm{C}(((0), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

Inferences:

9: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$ by

1: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

7: $\neg\ \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

10: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{C}((0, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$ by

1: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

8: $\neg\, 0 = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((0, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

11: $\neg\, 0 = x$ by

4: $\mathrm{C}((0, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

10: $\neg\, 0 = x \quad \vee \quad \neg\, \mathrm{C}((0, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$

12: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

5: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

6: $\neg\, \mathrm{S}(\mathrm{P}x) = x \;\vee\; \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \;\vee\; \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

13: $0 = y$ by

11: $\neg\, 0 = x$

0: $0 = x \quad \vee \quad 0 = y$

14: $\mathrm{S}(\mathrm{P}x) = x$ by

11: $\neg\, 0 = x$

3: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

15: $\mathrm{C}((y, 0, \mathrm{S}0)) = 0$ by

13: $0 = y$

2: $\mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad \neg\, 0 = y$

16: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

14: $\mathrm{S}(\mathrm{P}x) = x$

12: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

17: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

15: $\mathrm{C}((y, 0, \mathrm{S}0)) = 0$

9: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

18: QEA by

16: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

17: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$