Proof of Theorem 41a

The theorem to be proved is

$\mathrm{C}\big(x, 0, \mathrm{C}(y, 0, \mathrm{S}0)\big) = 0 \quad \rightarrow \quad x = 0 \quad \vee \quad y = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[(\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0))))) = (0)] \quad \& \quad [\neg\,(x) = (0)] \quad \& \quad [\neg\,(y) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$ from H:x:y

1: $\neg\, 0 = x$ from H:x:y

2: $\neg\, 0 = y$ from H:x:y

3: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad 0 = y$ from 36;y

4: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$ from 22;x

5: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ from 33;0;C((y,0,S0));Px

Equality substitutions:

6: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad 0 = \mathrm{C}((y, 0, \mathrm{S}0))$

7: $\neg\, \mathrm{S}(\mathrm{P}x) = x \ \vee \ \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \ \vee \ \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

Inferences:

8: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \mathrm{C}((y, 0, \mathrm{S}0)) = 0$ by
0: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0$
6: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = 0 \quad \vee \quad \neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

9: $\mathrm{S}(\mathrm{P}x) = x$ by
1: $\neg\, 0 = x$
4: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

10: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$ by
2: $\neg\, 0 = y$
3: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0 \quad \vee \quad 0 = y$

11: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

5: $\mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

7: $\neg\, \mathrm{S}(\mathrm{P}x) = x \ \vee\ \neg\, \mathrm{C}((\mathrm{S}(\mathrm{P}x), 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \ \vee\ \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

12: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

9: $\mathrm{S}(\mathrm{P}x) = x$

11: $\neg\, \mathrm{S}(\mathrm{P}x) = x \quad \vee \quad \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

13: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$ by

10: $\neg\, \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

8: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0)) \quad \vee \quad \mathrm{C}((y, 0, \mathrm{S}0)) = 0$

14: QEA by

12: $\mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$

13: $\neg\, \mathrm{C}((x, 0, \mathrm{C}((y, 0, \mathrm{S}0)))) = \mathrm{C}((y, 0, \mathrm{S}0))$