Proof of Theorem 40

The theorem to be proved is

$C(x, 0, 0) = 0$

Suppose the theorem does not hold. Then, with the variables held fixed,

(H) $[[\neg\ (C((x, 0, 0))) = (0)]]$

Special cases of the hypothesis and previous results:

0: $\neg\ C((x, 0, 0)) = 0$ from H:x

1: $0 = x \quad \vee \quad S(Px) = x$ from 22;x

2: $C((0, 0, 0)) = 0$ from 33;0;0;Px

3: $C((S(Px), 0, 0)) = 0$ from 33;0;0;Px

Equality substitutions:

4: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, 0)) = 0 \quad \vee \quad C((x, 0, 0)) = 0$

5: $\neg\ x = 0 \quad \vee \quad C(((x), 0, 0)) = 0 \quad \vee \quad \neg\ C(((0), 0, 0)) = 0$

Inferences:

6: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, 0)) = 0$ by
0: $\neg\ C((x, 0, 0)) = 0$
4: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, 0)) = 0 \quad \vee \quad C((x, 0, 0)) = 0$

7: $\neg\ 0 = x \quad \vee \quad \neg\ C((0, 0, 0)) = 0$ by
0: $\neg\ C((x, 0, 0)) = 0$
5: $\neg\ 0 = x \quad \vee \quad C((x, 0, 0)) = 0 \quad \vee \quad \neg\ C((0, 0, 0)) = 0$

8: $\neg\ 0 = x$ by
2: $C((0, 0, 0)) = 0$
7: $\neg\ 0 = x \quad \vee \quad \neg\ C((0, 0, 0)) = 0$

9: $\neg\ S(Px) = x$ by
3: $C((S(Px), 0, 0)) = 0$
6: $\neg\ S(Px) = x \quad \vee \quad \neg\ C((S(Px), 0, 0)) = 0$

10: $\mathrm{S}(\mathrm{P}x) = x$ by

8: $\neg\ 0 = x$

1: $0 = x \quad \vee \quad \mathrm{S}(\mathrm{P}x) = x$

11: QEA by

9: $\neg\ \mathrm{S}(\mathrm{P}x) = x$

10: $\mathrm{S}(\mathrm{P}x) = x$